SENSORLESS CONTROL OF BRUSHLESS PERMANENT MAGNET MOTORS

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A thesis submitted in partial fulfilment of the requirements of the University of Bolton for the degree of Doctor of Philosophy

This research programme was carried out in collaboration with South Westphalia University of Applied Sciences Department of Electrical Energy Technology Soest, Germany

December 2013
Declaration

I declare that I have developed and written the enclosed thesis entitled, “Sensorless Control of Brushless Permanent Magnet Motors,” by myself and have not used sources or means without declaration in the text. Any thoughts or quotations which are inferred from these sources are clearly marked.

This thesis was not submitted in the same or in a substantially similar version, not even portion of the work, to any authority to achieve any other qualification.

December 2013,

Chawanakorn Mantala
Abstract

In this thesis, a sensorless control method of permanent magnet synchronous machines (PMSMs), whose machine neutral points are accessible, for all speeds and at standstill is proposed, researched and developed. The sensorless method is called Direct Flux Control (DFC). The different voltages between a machine neutral point and an artificial neutral point are required for the DFC method. These voltages are used to extract flux linkage signals as voltage signals, which are necessary to approximate electrical rotor positions by manipulating the flux linkage signals. The DFC method is a continuous exciting method and based on an asymmetry characteristic and machine saliencies.

The DFC method is validated by implementing on both software and hardware implementation. A cooperative simulation with Simplorer for the driving circuit and programming the DFC and Maxwell for doing finite element analysis with the machine design is selected as the software simulation environment. The machine model and the DFC method are validated and implemented. Moreover, the influences of different machine structures are also investigated in order to improve the quality of the measured voltages.

The hardware implementation has been employed on two test benches, i.e. for small machines and for big machines. Both test benches use a TriCore PXROS microcontroller platform to implement the DFC method. There are several PMSMs, both salient poles and non-salient poles, which are used to validate the DFC method. The flux linkage signals are also analyzed. The approximation of the flux linkage signal is derived and proposed. A technique to remove the uncertainty of the calculated electrical rotor position based on the inductance characteristics has been found and implemented. The electrical rotor position estimation method has been developed based on the found flux linkage signal approximation function and analyzed by comparing with other calculation techniques.

Moreover, the calculated electrical rotor position is taken into account to either assure or show the relation with the exact rotor position by testing on the hardware environment. The closed loop speed sensorless control of PMSMs with DFC is
Abstract

presented and executed by using the assured calculated electrical rotor position to perform the DFC capability.

This thesis has been done in the Electric Machines, Drives and Power Electronics Laboratory, South Westphalia University of Applied Sciences, Soest, Germany.

Keywords: Direct Flux Control (DFC), Permanent Magnet Synchronous Machine (PMSM), Sensorless Control, Machine Saliencies, Field Oriented Control (FOC).
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December 2013,

Chawanakorn Mantala
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\( p \) Subscript stands for the phase \( U, V, W \)

\( \text{raw} \) Subscript stands for raw information

\( s \) Subscript stands for strengthening signal

\( w \) Subscript stands for weakening signal

\( ' \) Superscript stands for the first derivative by time

\( \alpha \) Electrical rotor position \([\text{rad}, \degree]\)

\( \alpha_{\text{cal}} \) Calculated electrical rotor position \([\text{rad}, \degree]\)

\( \alpha_{m} \) Summation of calculated electrical rotor position and electrical correction angle \([\text{rad}, \degree]\)

\( \alpha_{k} \) Electrical correction angle \([\text{rad}, \degree]\)

\( \alpha_{k,\text{cal}} \) Estimated mechanical rotor position \([\text{rad}, \degree]\)

\( \alpha_{\text{cal},m} \) Calculated electrical rotor position by \( m \) method \([\text{rad}, \degree]\)

\( (\alpha, \beta) \) Stationary frame

\( \xi \) Ratio between \( L_{q} \) and \( L_{y} \)

\( \Psi_{\alpha} \) Flux in \( \alpha \) axis \([\text{Wb}, \text{Vs}]\)

\( \Psi_{\beta} \) Flux in \( \beta \) axis \([\text{Wb}, \text{Vs}]\)

\( \Psi_{d} \) Flux in \( d \) axis \([\text{Wb}, \text{Vs}]\)

\( \Psi_{q} \) Flux in \( q \) axis \([\text{Wb}, \text{Vs}]\)
<table>
<thead>
<tr>
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<th>Description</th>
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<tr>
<td>( \Psi^* )</td>
<td>Coupled flux linkage of other phases including the permanent rotor flux ([\text{Wb} , \text{Vs}])</td>
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<td>( \Psi_p )</td>
<td>Resultant flux linkage of phase ( p ) ([\text{Wb} , \text{Vs}])</td>
</tr>
<tr>
<td>( \Psi_r )</td>
<td>Rotor flux ([\text{Wb} , \text{Vs}])</td>
</tr>
<tr>
<td>( \Psi_s )</td>
<td>Stator flux ([\text{Wb} , \text{Vs}])</td>
</tr>
<tr>
<td>( \Psi_t )</td>
<td>Total flux ([\text{Wb} , \text{Vs}])</td>
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<tr>
<td>( \tau )</td>
<td>Time constant of RL circuit ([\text{s}])</td>
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<td>( \tau_d )</td>
<td>Time constant in ( d ) axis ([\text{s}])</td>
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<td>( \tau_s )</td>
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<td>Sampling frequency ([\text{rad/s}])</td>
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<td>( \omega_p )</td>
<td>Electrical frequency ([\text{rad/s}])</td>
</tr>
<tr>
<td>( \omega_k )</td>
<td>Mechanical frequency ([\text{rad/s}])</td>
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<tr>
<td>( (d,q) )</td>
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<tr>
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<td>Measured current in ( d ) axis ([\text{A}])</td>
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<td>( I_q )</td>
<td>Current in ( q ) axis ([\text{A}])</td>
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<td>$I_{q,m}$</td>
<td>Measured current in $q$ axis [A]</td>
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<td>$I_{q,s}$</td>
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<td>$I_{sum}$</td>
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<td>Inductance in $d$ axis while weakening the magnetic field [H]</td>
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<tr>
<td>$L_x$</td>
<td>Summation of $L_d$ and $L_q$ [H]</td>
</tr>
<tr>
<td>$L_y$</td>
<td>Difference between $L_d$ and $L_q$ [H]</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Desired mechanical speed [rpm, s$^{-1}$]</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Calculated mechanical speed [rpm, s$^{-1}$]</td>
</tr>
<tr>
<td>$PL_p$</td>
<td>Position signal by using phase inductance of phase $p$ [ V$^{1/2}$]</td>
</tr>
<tr>
<td>$P_M$</td>
<td>Maximum duty cycle of PWM unit [%]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>POS</td>
<td>Counter signal of incremental encoder</td>
</tr>
<tr>
<td>$p_R$</td>
<td>Number of permanent magnet pole pairs</td>
</tr>
<tr>
<td>$R_p$</td>
<td>Resistance of phase $p$ [Ω]</td>
</tr>
<tr>
<td>$R_{ANp}$</td>
<td>Resistance of the artificial neutral point circuit of phase $p$ [Ω]</td>
</tr>
<tr>
<td>$t$</td>
<td>Time [s]</td>
</tr>
<tr>
<td>$t_m$</td>
<td>Measuring time after switching on pulse [s]</td>
</tr>
<tr>
<td>$t_o$</td>
<td>Switch on time the next pulse [s]</td>
</tr>
<tr>
<td>$T_{cog}$</td>
<td>Cogging torque [Nm]</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Torque load [Nm]</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Machine Torque [Nm]</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sampling time of data processing [s]</td>
</tr>
<tr>
<td>$u^*$</td>
<td>Approximated flux linkage signal of phase $U$ [V]</td>
</tr>
<tr>
<td>$u$</td>
<td>Flux linkage signal or DFC signal of phase $U$ [V]</td>
</tr>
<tr>
<td>$U$</td>
<td>Phase $U$</td>
</tr>
<tr>
<td>$v$</td>
<td>Flux linkage signal or DFC signal of phase $V$ [V]</td>
</tr>
<tr>
<td>$V$</td>
<td>Phase $V$</td>
</tr>
<tr>
<td>$V_{AN}$</td>
<td>Voltage at the artificial neutral point [V]</td>
</tr>
<tr>
<td>$V_{DC}$</td>
<td>DC link or DC bus voltage [V]</td>
</tr>
<tr>
<td>$V_N$</td>
<td>Voltage at the machine neutral point [V]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$V_{NAN}$</td>
<td>Different voltage between the machine neutral point and the artificial neutral point [V]</td>
</tr>
<tr>
<td>$V_{offset}$</td>
<td>DC offset for phase inductances calculation [V]</td>
</tr>
<tr>
<td>$V_{zSV}$</td>
<td>Zero sequence voltage [V]</td>
</tr>
<tr>
<td>$w$</td>
<td>Flux linkage signal or DFC signal of phase $W$ [V]</td>
</tr>
<tr>
<td>$W$</td>
<td>Phase $W$</td>
</tr>
</tbody>
</table>
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADC</td>
<td>Analog to digital converter</td>
</tr>
<tr>
<td>BPF</td>
<td>Band pass filter</td>
</tr>
<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>DFC</td>
<td>Direct Flux Control</td>
</tr>
<tr>
<td>ELO</td>
<td>Extended Luenberger observer</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman filter</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive force</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic interference</td>
</tr>
<tr>
<td>FADC</td>
<td>Very fast analog to digital converter</td>
</tr>
<tr>
<td>FOC</td>
<td>Field oriented control</td>
</tr>
<tr>
<td>HPF</td>
<td>High pass filter</td>
</tr>
<tr>
<td>INFORM</td>
<td>Indirect Flux detection by online Reactance Measurement</td>
</tr>
<tr>
<td>LCM</td>
<td>Least common multiple number</td>
</tr>
<tr>
<td>LPF</td>
<td>Low pass filter</td>
</tr>
<tr>
<td>PF</td>
<td>Position calculation by using the trigonometric relation of the flux linkage signals</td>
</tr>
<tr>
<td>Ph</td>
<td>Position calculation by using the highest value</td>
</tr>
<tr>
<td>Pl</td>
<td>Position calculation by using the lowest value</td>
</tr>
<tr>
<td>PL</td>
<td>Position calculation by using the relation of phase inductance position signals</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase locked loop</td>
</tr>
</tbody>
</table>
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMSM</td>
<td>Permanent magnet synchronous machine</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse width modulation</td>
</tr>
<tr>
<td>rpm</td>
<td>Round per minute</td>
</tr>
<tr>
<td>SMO</td>
<td>Sliding mode observer</td>
</tr>
<tr>
<td>TTL</td>
<td>Transistor transistor logic</td>
</tr>
<tr>
<td>VTA</td>
<td>Voltage time area</td>
</tr>
<tr>
<td>ZSV</td>
<td>Zero sequence voltage</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 Problem Statement and Motivation

Millions of small DC motors are produced everyday in order to be utilized in a wide range of applications, e.g. coffee machines, automated teller machines (ATM) and especially in mechatronic drive systems in automobiles. Advantages of these motors are their simple design and the relatively low price. A significant disadvantage is the required space, due to the installation of the mechanical commutator and carbon brushes, which also result in low efficiency and high maintenance.

Due to the mentioned significant disadvantage, a brushless motor has been selected instead of those motors. The brushless motors can be called as electronically commutated motors, which can be divided into two types, brushless DC motor (BLDC) and brushless AC motor. The difference between both types is the characteristic of the induced electromotive force (EMF) or back EMF. The BLDC machine has a trapezoidal back EMF, and the drive strategy is required to keep the back EMF and the currents as DC signals by using the drive technology with control topologies. The BLDC becomes popular because of its control simplicity. However, it cannot appropriately work at low speeds and the torque is actually less smooth as it is with a brushless AC motor.

For the brushless AC motors, there are also many types, e.g. asynchronous induction motor, reluctance motor, permanent magnet synchronous motor (PMSM). The reluctance motor is usually used as fan and pump. The drawbacks are the strong noise sound and the cogging torque behavior. The induction motor is the cheapest motor, it is also popular in general applications e.g. ventilator. However, it has to be perfectly designed with the control scheme and considering about the load, when it has to deal with complicated tasks.

Consequently, the PMSM is the solution of the recent motor technologies. It has the simple structure as the synchronous machines with the permanent magnet rotors. PMSM can be perfectly run, having very high efficiency and being very robust, when connected through an efficient system with power electronics devices and modern microprocessor control technology. PMSMs have been applied to use in various fields, e.g. renewable energy as windmill, medical equipment, pump, segway,
washing machine. Particularly, the automobile industry will increasingly use electric systems as a replacement for hydraulic or pneumatic systems because of their limited maneuverability in applications of the engine management, such as for the electric power steering in which brushless drives are currently implemented. Thus, the brushless permanent magnet motors, PMSMs, are taken into account in this thesis.

However, a major disadvantage of these machines is the requirement of a sensor system to detect the rotor position. This sensor requiring extra space and cabling will lead to additional costs. A diagram of a PMSM closed loop control based on the field oriented control (FOC) is depicted in Fig. 1.1. It is shown that the speed and position can be usually found by traditional measurements, such as resolvers and absolute encoders.

Consequently, the sensorless control method should be a solution to solve this problem. In fact, sensorless methods have been implemented to perform in some applications, e.g. pumps and fans, but the methods cannot work for the whole range of the machine speed. This means that a minimum speed is required, whilst it will not work in the standstill condition. Hence, the solution to overcome these problems is a crucial challenge task.

Fig. 1.1: FOC of Permanent Magnet Synchronous Machine

1.2 Research Aims

The aim of the research is the validation and development of sensorless control of permanent magnet synchronous machine or brushless permanent magnet motors by a
sensorless method based on the patents [1 – 3]. The mentioned method is called Direct Flux Control (DFC). The objectives can be listed as follows.

- To develop and validate the Direct Flux Control (DFC) method both in software and hardware implementation.
- To implement and investigate the DFC method in order to work with different permanent magnet synchronous motors.
- To analyze the DFC method characteristics and its restrictions.
- To derive the DFC method and analyze which motor properties influence and relate to the DFC method.
- To achieve sensorless control of permanent magnet synchronous motors for all speeds by Direct Flux Control (DFC).

1.3 Research Contribution

Currently, no existing technique for the sensorless rotor position detection of permanent magnet synchronous machines works for all speeds with several restrictions. Firstly, the sensorless DFC–method does not need machine parameters which is a significant advantage compared to existing observer methods. Secondly, the driving system can be run without interrupting to inject any measurement sequence, different from other methods, e.g. INFORM. Next, it can forego pre- or self-commissioning of the machine to figure out machine electrical parameters, which relate to the anisotropy signal characteristics. Especially, the rotor position can be acquired for all speeds by using only few measured electrical values and is also compatible with other control strategies to achieve the control methods purposes.

Consequently, a number of new contributions are to reduce the listed restrictions, which are developed and implemented under this research by using DFC. For that reason, the DFC method is researched and developed in order to fulfill the mentioned requirements.
Thus, there are several scientific contributions, which are based on the DFC method and contributed in this thesis as stated below.

- A flux linkage signal approximation function of DFC is proposed.
- A technique to remove the uncertainty of north and south pole is proposed.
- An elimination strategy for the fourth harmonic of the flux linkage signal is proposed.
- Different rotor position calculation methods are investigated, analyzed and proposed.
- All aspects of DFC, e.g. restrictions, analysis and influences, are researched and elaborated.
- The DFC method is applied to work with a control loop.

1.4 Dissertation Structure

Chapter 1 presents the introduction of research. Problem statement and motivation, research aims and contribution are given. The dissertation structure is stated.

Chapter 2 presents the state of the art, which describes the recent technologies of the sensorless control methods of the permanent magnet synchronous machines.

Chapter 3 explains the principle of the DFC method. Both software simulation and hardware implementation to validate the DFC method with PMSMs are explained and achieved. The experimental results are given and also discussed.

Chapter 4 analyzes on the DFC method. The machine design structures and the measured signals in the real time system have been taken into account to improve and determine the sensorless method.

Chapter 5 proposes the flux linkage signal approximation function, which is derived by using the phase inductance characteristics in stator frame with the DFC conditions. The technique to remove the uncertainty of north and south poles is also proposed based on the magnetic reluctance behavior.
Chapter 6 uses the found flux linkage signal approximation to develop the electrical rotor position calculation methods. All methods have been executed both in software and hardware environments. The proper methods have been found. The experiments to assure the calculated rotor position as the exact rotor position have been achieved.

Chapter 7 applies the DFC method to implement the closed loop sensorless speed control. The motor is controlled by the field oriented control (FOC) approach. The closed loop speed control structure design is also elucidated. The experimental results are given and also analyzed.

Chapter 8 concludes all aspects of this dissertation including future works, which can simply be applied in the further investigations.
2 State of the Art

The sensorless control is the combination between rotor position estimation techniques and control strategies to control and drive machines without mechanical sensors to measure the rotor position, e.g. resolvers and absolute encoders.

The main purpose of machine control strategies is to reach a desired speed with maximum torque when the machine is driven. Consequently, the rotor position is the most required value, because it can be used to calculate the rotor speed and to get a perpendicular angle between an available machine magnetic field and a resultant machine current. One of the most well known methods is a field oriented control (FOC) strategy, which is also implemented in this dissertation.

According to the PMSM rotor position estimation methods, they have been researched and developed for a few decades. Recently, the position estimation or sensorless methods can be divided into two approaches, i.e. back electromotive force (EMF) and machine saliencies.

2.1 Back EMF Based Methods

The back EMF based sensorless methods are typically achieved by two ways, i.e. using the relation between the magnitude and the frequency of applied input voltages to steer a machine up to a minimum speed and then use the back EMF zero-crossings for commutation, and utilizing observers to imitate machines behaviors and estimate the machine state. The observer is based on the state space system, which is a dynamic system whose characteristics are somewhat free to be determined by the designer and it is through its introduction that dynamics enter the overall two-phase design procedure, i.e. the design of the control law assuming the state is available and the design of a system that produces an approximation to the state vector, when the entire state is unavailable [4].

The observers can be mainly divided into 3 types, i.e. deterministic observer [5] e.g. extended Luenberger observer (ELO), probabilistic observer [6] e.g. extended Kalman filter (EKF), and nonlinear observer [7] e.g. sliding mode observer (SMO), which are shown in Fig 2.1, 2.2, and 2.3, respectively.
The ELO and the EKF can use the same state space model in order to implement, but the difference is the estimation technique. Moreover, the EKF can properly deal with further conditions such as unchanged speed and unchanged load conditions, but the ELO cannot work. This is because the EKF is a recursive filter and based on a stochastic algorithm, which means that the EKF can estimate although the assumed condition is false. However, both estimation solutions are extended from linear system solutions to apply with a nonlinear system. Therefore, a nonlinear observer as the sliding mode observer (SMO) can be another solution to estimate the needed values. A sliding function is generally included in a current observer as in Fig. 2.3, which leads to have a low pass filter (LPF) in order to filter a chattering output from the current observer. Moreover, other functions are proposed to be used as the sliding function as represented in Fig. 2.4 to decrease the chattering characteristic.
Overall, the limitations of these sensorless methods are sensitivity to machine parameters and they cannot work at low speeds and standstill. They can only work when the driven machine back EMF level is high enough (10 – 20% of rated machine voltage) and the machine model parameters must be correct [8].

Therefore, the back EMF based methods have been improved to deal with the mentioned obstacles. A few electrical signals in the control scheme, the output voltages of the current controllers, are selected to be inputs of the sensorless methods. The machine parameters become less sensitive and the operating speed regions (low and high speeds) are divided by applying a loop recovery technique to drive with vector control at low speeds and smooth transition between regions in [9]. A low cost sensorless control algorithm by reducing equipment with high dynamic performance is also implemented in [10]. However, a back EMF based method, which can work for all speeds, is unavailable.
2.2 Machine Saliencies Based Methods

The machine saliencies based sensorless methods ([11 – 24]) are considered to solve the difficulty to estimate the rotor position from standstill to high speeds as the full speed range. In the field of electrical machines, these methods use high frequency signals to excite the machines based on asymmetry conditions. The rotor position can be estimated by using the idea that the inductances dependent position consists of a single harmonic. In point of fact, the rotor position extraction methods are based on measured signals of each method with the standard electrical machine model, especially the inductance model for high frequency excitation.

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>Excitation Method</th>
<th></th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Continuous</td>
<td>Discontinuous</td>
<td></td>
</tr>
<tr>
<td>Periodic</td>
<td>PWM</td>
<td>PWM</td>
<td></td>
</tr>
<tr>
<td>Current Sensor</td>
<td>[14 – 16], [20]</td>
<td>[21– 23]</td>
<td>[11 – 13]</td>
</tr>
<tr>
<td>Voltage Sensor</td>
<td>[17], [20]</td>
<td>[18 – 19], [20]</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Categorization of machine saliencies based sensorless methods (refer to the numbers of references)

Concerning the high frequency excitation methods, they can be divided into three excitation types, i.e. continuous, discontinuous and other methods as represented in Table 2.1. The measured signals, i.e. current and voltage signals, depend on the methods. The difference between periodic and pulse width modulation is the excitation way. The periodic excitation is to inject a periodic carrier signal and encode the rotor position by using the magnitude or phase information of the measured signals. For the PWM excitation, the PWM signal forms and switching states of the
inverter are modified to acquire the particular information, which can be used to calculate the rotor position.

A discontinuous method is INFORM [11], which can work at standstill and low speeds. Measurement sequences are periodically applied by setting the PWM patterns and interrupting the driving system. The measured currents are used to approximate the rotor position. For instance, the measurement sequences of phase $U$ are depicted in Fig. 2.5. The test voltage space vectors in the stationary frame i.e. $-u$ and $+u$ are applied and the phase current is measured. The resultant voltage of the applied sequences is zero based on the pulse width modulation (PWM) switching patterns. In order to estimate the electrical rotor position, two phase currents are required. Thus, the driving system is interrupted in a short time period ($T_{INF}$) to measure the phase current as shown in a time diagram in Fig. 2.6. The measurement sequences and the calculation are improved in [12] and [13]. Due to the discontinuity and the measuring difficulty, INFORM cannot work for high speeds.

![Fig. 2.5: Measurement sequences of phase $U$ [12]](image1)

![Fig. 2.6: INFORM time diagram [12]](image2)
For periodic continuous excitation methods, both current and voltage sensors are utilized. A carrier voltage signal \( V_{(d,q),C} \) is injected to combine with an output of each current controller in the synchronous frame by modifying a regular control loop as represented in Fig. 2.7.

![Fig. 2.7: Carrier signal injection](image)

The carrier voltage signal generates a carrier current signal, which contains the information of the electrical rotor position, which is a negative-sequence carrier signal current. It can be acquired by filtering the measured currents of two out of three phases with a high pass filter (HPF). The negative-sequence carrier signal current is used to extract the position by working with a tracking observer [14]. It is also developed by an improved phase locked loop (PLL) type estimator to work with the back EMF estimation [15]. A control harmonic scheme of the low frequency of the negative-sequence is also analyzed in [16]. However, the measured negative-sequence carrier signal is generated by the voltage carrier signal. As a basic relation between voltage and current in alternating current systems, the current is reduced when the voltage is constant and the frequency is higher.

Due to the mentioned problem, a zero sequence voltage signal as the summation of three phase voltages is used. The zero sequence carried signal measurement method is investigated and developed in [17]. The optimized method, using a voltage sensor and less noise from a DC link voltage circuit, is to measure the zero sequence carried signal by using different voltages between a machine neutral point and an artificial neutral point as depicted in Fig. 2.8. The electrical rotor position information, which is included in the zero sequence carried signal, can be extracted by filtering the signal and using approximation functions whose parameters are found by experiments. Nevertheless, the machine currents consist of harmonics caused by the carrier signal...
injection. Moreover, the non-ideal inverter characteristics and high frequency incident e.g. grounding also influence the method. Hence, another option in [18] and [19] can be selected to work instead. The zero sequence voltage signal of each phase is measured while the PWM unit is switching at particular states. The measured signals are obtained by sampling at the inverter terminals with the cabling of the machine neutral line. It is worth to mention that the methods in [14 – 19] have been fundamentally compared in [20]. However, a pre- or self-commissioning has to be done to design the spatial filter and to obtain the machine necessary parameters, which are included in the rotor position approximation functions.

Fig. 2.8: Zero sequence voltage signal measuring scheme

Fig. 2.9: Current derivatives measuring scheme
Besides, the different measured current derivatives between the PWM switching states are used to extract the rotor anisotropy signals, from which the rotor position can be calculated as in [21 – 23]. The current derivatives measuring scheme is represented in Fig. 2.9. The PWM switching particular states are also required as same as in [18 – 19] to measure the current derivatives. The modified pulse patterns are achieved by remaining the voltage time area (VTA) to be the same as the required voltage space vector. Notwithstanding, the mentioned processes, e.g. the pre-commissioning to attain the machine parameters, cannot be foregone.

It is worth to mention that a sensorless method without a self-commissioning process is also shown in [24], but the rotor position can be found only at standstill by measuring machine line to neutral voltage while injecting a high frequency voltage signal to another phase with either a machine neutral point or an artificial neutral point as depicted in Fig. 2.10 (a) and (b), correspondingly. All three phase machine line to neutral voltages are measured and converted into the stationary frame signals to estimate the rotor position by the trigonometric relations. Moreover, each motor has an optimal injected frequency. It can be found by varying the injected frequency and selecting the frequency, which can generate the maximum different voltage between the measured voltages.

![Diagram](image)

a) Accessible machine neutral point  
b) Inaccessible machine neutral point

Fig. 2.10: Line to neutral voltage measuring scheme
2.3 Summary

All in all, it is shown that there is no existing sensorless method, which can estimate the electrical rotor position, which machine parameters are unnecessary to be known. Moreover, in order to implement for a wide range speed and at standstill, the continuous excitation method is required. Thus, it is a crucial task to implement and develop a sensorless method, which can work for all speeds and sacrifice all mentioned limitations. As a result, the DFC method is implemented, developed and researched in this dissertation in order to be the solution as the most recent sensorless method technology.
3 Principle of Direct Flux Control

In this chapter, the explanation of DFC is shown and the DFC method is implemented on both software and hardware environments. The experimental setup and experimental results of each environment are described and illustrated with discussion, respectively.

3.1 Direct Flux Control (DFC)

The DFC method is based on [1 – 3]. The method can be divided into three parts, theoretical background, flux linkage extraction, and electrical rotor position calculation. Each part is explained as follows.

3.1.1 Theoretical Background

The machine phase voltage equation can be stated as in (3.1).

\[
V_p = I_p R_p + \frac{d\Psi_p}{dt}
\]  

(3.1)

Where subscript \( p \) is the phase, \( \Psi \) is the resultant flux linkage of phase \( p \), which can be distributed in (3.2).

\[
\Psi_p = L_{\alpha\beta} I_p + \Psi^*
\]  

(3.2)

\( \Psi^* \) is the coupled flux linkage of other phases including the permanent rotor flux, which generates the back EMF. \( L_{\alpha\beta} \) is the machine phase inductance. Hence, the first derivative of \( \Psi_p \) can be calculated and rearranged in (3.3 – 3.5).

\[
\frac{d\Psi_p}{dt} = L_{\alpha\beta} \frac{dI_p}{dt} + I_p \frac{dL_{\alpha\beta}}{dt} + \frac{d\Psi^*}{dt}
\]  

(3.3)

\[
\frac{d\Psi_p}{dt} = L_{\alpha\beta} \frac{dI_p}{dt} + \frac{d\Psi^*}{dt}
\]  

(3.4)

\[
L_{\alpha\beta} = L_{\alpha\beta} + I_p \frac{dL_{\alpha\beta}}{dt}
\]  

(3.5)
Therefore, the phase voltage machine equation can be concluded as in (3.6), which is the main equation to implement the DFC method. $L_{\sigma^p}$ is also the key value of DFC, which is described in the next part.

$$V_p = I_p R_p + L_{\sigma^p} \frac{dI_p}{dt} + \frac{d\Psi^*}{dt}$$

(3.6)

### 3.1.2 Flux Linkage Extraction

The DFC method uses flux linkage signals as voltage signals to estimate the electrical rotor position. Thus, the flux linkage signal is extracted by utilizing the different voltage between the three phase machine neutral point and an artificial neutral point ($V_{NAN}$) as displayed in Fig. 3.1. The resistances of the artificial neutral point circuit are much larger than the machine phase resistances ($R_p \ll R_{ANp}$), which leads to have no current flow in the artificial neutral point circuit. Thus, the currents in the artificial neutral point circuit can be neglected.

![Fig. 3.1: Flux linkage extraction measuring scheme](image)

In addition, the DFC method is a continuous excitation method by modifying pulse width modulation (PWM) patterns, which are inputs of the inverter. The PWM pattern is based on the voltage vector and the modulation technique. For the DFC method, every modulation technique can be used, but the restriction is that the pulses must not be switched on at the same time with a small delay to obtain the measurable $V_{NAN}$. For instance, the pulse pattern of DFC as illustrated in Fig. 3.2 is applied. There are four states, i.e. 0 to 4. Each state equivalent circuit is depicted in Fig. 3.3.
From state 0 to 4, the three phase machine voltage equation of each state can be listed in (3.7) to (3.10), correspondingly.

\[
V_{NAN_q} = I_U R_U + L_{\sigma U} \frac{dI_U}{dt} + \frac{d\Psi^*}{dt}
\]

\[
V_{NAN_q} = I_V R_V + L_{\sigma V} \frac{dI_V}{dt} + \frac{d\Psi^*}{dt}
\]: State 0

\[
V_{NAN_q} = I_W R_W + L_{\sigma W} \frac{dI_W}{dt} + \frac{d\Psi^*}{dt}
\]

\[
V_{NAN_i} - V_{DC} = I_U R_U + L_{\sigma U} \frac{dI_U}{dt} + \frac{d\Psi^*}{dt}
\]

\[
V_{NAN_i} = I_V R_V + L_{\sigma V} \frac{dI_V}{dt} + \frac{d\Psi^*}{dt}
\]: State 1

\[
V_{NAN_i} = I_W R_W + L_{\sigma W} \frac{dI_W}{dt} + \frac{d\Psi^*}{dt}
\]
Principle of Direct Flux Control

\[ V_{NAN} - V_{DC} = I_U R_U + L_{\sigma U} \frac{dI_U}{dt} + \frac{d\Psi^*}{dt} \]

\[ V_{NAN} - V_{DC} = I_V R_V + L_{\sigma V} \frac{dI_V}{dt} + \frac{d\Psi^*}{dt} \quad \text{: State 2} \quad (3.9) \]

\[ V_{NAN} = I_W R_W + L_{\sigma W} \frac{dI_W}{dt} + \frac{d\Psi^*}{dt} \]

\[ V_{NAN} - V_{DC} = I_U R_U + L_{\sigma U} \frac{dI_U}{dt} + \frac{d\Psi^*}{dt} \]

\[ V_{NAN} - V_{DC} = I_V R_V + L_{\sigma V} \frac{dI_V}{dt} + \frac{d\Psi^*}{dt} \quad \text{: State 3} \quad (3.10) \]

\[ V_{NAN} - V_{DC} = I_W R_W + L_{\sigma W} \frac{dI_W}{dt} + \frac{d\Psi^*}{dt} \]

Subsequently, the flux linkage signals can be extracted based on the high frequency excitation model conditions as in [11 – 24], which are that both the resistive terms and the back EMF can be neglected and energy storages are not changed. There are two more conditions of three phase systems as in (3.11) and (3.12).

\[ I_U + I_V + I_W = 0 \quad (3.11) \]

\[ \frac{dI_U}{dt} + \frac{dI_V}{dt} + \frac{dI_W}{dt} = 0 \quad (3.12) \]

After all conditions are taken into account, (3.7) to (3.10) can be rearranged in (3.13) to (3.16), respectively. Then, the flux linkage signals of \( U, V, \) and \( W \) are \( u, v, \) and \( w, \) can be simply found as voltage signals by subtracting \( V_{NAN} \) between two switching states. \( u, v, \) and \( w \) are represented in (3.17).

\[ V_{NAN} = \left( \frac{I_U R_U}{L_{\sigma U}} + \frac{I_V R_V}{L_{\sigma V}} + \frac{I_W R_W}{L_{\sigma W}} \right) + \left( \frac{d\Psi^*}{dt} \frac{dt}{L_{\sigma U}} + \frac{d\Psi^*}{dt} \frac{dt}{L_{\sigma V}} + \frac{d\Psi^*}{dt} \frac{dt}{L_{\sigma W}} \right) \]

\[ \left( \frac{1}{L_{\sigma U}} + \frac{1}{L_{\sigma V}} + \frac{1}{L_{\sigma W}} \right) \]

(3.13)
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\[ V_{\text{NAN}_1} = \frac{V_{\text{DC}}}{L_{\sigma^U}^*} + \frac{(I_{I^R} R_{U}^* + I_{I^R} R_{V}^* + I_{I^R} R_{W}^*)}{L_{\sigma^U}^*} + \frac{d\Psi^*}{dt} + \frac{d\Psi^*}{dt} + \frac{d\Psi^*}{dt} \]

\[ V_{\text{NAN}_1} = \frac{V_{\text{DC}} + V_{\text{DC}} + + V_{\text{DC}}}{L_{\sigma^U}^*} + \frac{(I_{I^R} R_{U}^* + I_{I^R} R_{V}^* + I_{I^R} R_{W}^*)}{L_{\sigma^V}^*} + \frac{d\Psi^*}{dt} + \frac{d\Psi^*}{dt} + \frac{d\Psi^*}{dt} \]

\[ V_{\text{NAN}_1} = \frac{V_{\text{DC}}}{L_{\sigma^U}^*} + \frac{V_{\text{DC}} + + V_{\text{DC}}}{L_{\sigma^V}^*} + \frac{d\Psi^*}{dt} + \frac{d\Psi^*}{dt} + \frac{d\Psi^*}{dt} \]

\[ u = V_{\text{NAN}_1} - V_{\text{NAN}_0} = \frac{1}{L_{\sigma^U}^*} \] \[
\frac{L_{\sigma^U}^* + L_{\sigma^U}^* + L_{\sigma^U}^*}{V_{\text{DC}}} \]

\[ v = V_{\text{NAN}_2} - V_{\text{NAN}_1} = \frac{1}{L_{\sigma^V}^*} \]

\[ w = V_{\text{NAN}_1} - V_{\text{NAN}_2} = \frac{1}{L_{\sigma^W}^*} \]

It is noteworthy that there are six possibilities of each phase to obtain each flux linkage signal. This is because it depends on the applied voltage vector to the inverter. For instance, the six cases to obtain \( u \) are shown in (3.18). Where \( V_{\text{NAN} (PWM (U), PWM (V), PWM (W))} \) is \( V_{\text{NAN}} \) at the state of PWM unit.
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\[
\begin{align*}
\mathbf{u} &= \left[
\begin{array}{c}
V_{\text{NNN}(1,0,0)} - V_{\text{NNN}(0,0,0)} \\
V_{\text{NNN}(1,1,0)} - V_{\text{NNN}(0,1,0)} \\
V_{\text{NNN}(1,1,1)} - V_{\text{NNN}(0,1,1)} \\
-(V_{\text{NNN}(0,0,0)} - V_{\text{NNN}(1,0,0)}) \\
-(V_{\text{NNN}(0,1,0)} - V_{\text{NNN}(1,1,0)}) \\
-(V_{\text{NNN}(0,1,1)} - V_{\text{NNN}(1,1,1)})
\end{array}
\right] \\
&= (3.18)
\end{align*}
\]

3.1.3 Electrical Rotor Position Calculation

For sensorless methods based on machine saliencies with high frequency excitation in [11 – 24], there is more than one signal type to estimate the rotor position. The DFC method uses the extracted flux linkage signals to calculate the electrical rotor position. The flux linkage signals are assumed to be a periodical signal with a dominant frequency in order to keep away from ripple flux characteristics in the synchronous frame as analyzed in [25]. The assumed flux linkage signals are second harmonic signals as depicted in Fig. 3.4.

Consequently, the electrical rotor position ($\alpha_{\text{calc}}$) can be found by the relation of each flux linkage signal as in (3.19) and depicted in Fig. 3.5.
Fig. 3.5: Calculated electrical rotor position

Fig. 3.5 shows that these three repetitive cases which are in the range of 0 to 360 degrees, can be manipulated to be 0 to 180 degrees ($\pi$ rad) in one period as calculated in (3.20). The manipulated electrical rotor position is normalized and depicted in Fig. 3.6. Finally, two periods of the manipulated position are combined to be a period of the calculated electrical rotor position ($\alpha_{cal}$) in the range of 0 to $2\pi$ rad (360 degrees) as plotted in Fig. 3.7. Even though the calculated electrical rotor position is in the regular range, an uncertainty of magnet poles ($\pm$180 degree) has not been taken into account. The uncertainty can lead to drive the machine in an opposite direction.

$$
\alpha_{cal} = \begin{cases} 
\frac{v-w}{u} : (u \leq v) \cap (u \leq w) \\
\frac{w-u}{v} : (v \leq u) \cap (v \leq w) \\
\frac{u-v}{w} : (w \leq u) \cap (w \leq v)
\end{cases} \quad (3.19)
$$

$$
\alpha_{cal} = \begin{cases} 
\frac{v-w}{u} \cdot \frac{\pi}{18} + \frac{\pi}{2} : (u \leq v) \cap (u \leq w): \text{ case1} \\
\frac{w-u}{v} \cdot \frac{\pi}{18} + \frac{5\pi}{6} : (v \leq u) \cap (v \leq w): \text{ case2} \\
\frac{u-v}{w} \cdot \frac{\pi}{18} + \frac{\pi}{6} : (w \leq u) \cap (w \leq v): \text{ case3}
\end{cases} \quad (3.20)
$$
3.2 DFC Implementation

As described in the previous section, the DFC method is executed by implementing both on software and hardware environments. There are three PMSMs, which are experimented. Each machine description is listed as follows.
a. PMSM1

PMSM1 is a small PMSM, has an out rotor with 12 permanent magnets and 9 stator teeth with a machine neutral point accessible. The electrical power of PMSM1 is 7.5 W, approximately. The PMSM1 is depicted in Fig. 3.8.

![PMSM1](image1)

Fig. 3.8: PMSM1

b. PMSM2

PMSM2 is a bigger motor than PMSM1, consists of an out rotor with 20 permanent magnet pole pairs (40 permanent magnets) and 54 stator teeth with a machine neutral point accessible. The blank area inside the motor as in Fig. 3.9, was used for a Hall sensor which has been removed. The PMSM2 power is 712 W, approximately.

![PMSM2](image2)

Fig. 3.9: PMSM2
c. PMSM3

PMSM3 has a different structure, when compared to PMSM1 and PMSM2. The magnets are buried inside the rotor, which is called non-salient poles PMSM or buried magnet PMSM. The power of PMSM3 is in the region of 3.7 kW. This machine has 9 stator teeth and 6 magnets.

![PMSM3](image)

Fig. 3.10: PMSM3

### 3.2.1 Software Implementation

Generally, there are two parts, i.e. a machine model and a drive circuit with programmable algorithms, which are required to do software simulation.

Consequently, Simulink is selected as a simulation environment to execute the DFC method with PMSM models. The machine models in Simulink are available only in synchronous frame \( (d,q) \) with constant phase inductance values and without the machine neutral point. Thus, a new PMSM model with nonlinear phase inductance characteristics and the neutral point accessibility has to be designed. In [26] and [27], the saturation of the lamination core has been implemented into machine models, which are represented by coefficients instead of the nonlinear magnetization (BH) curve, whose inductances can be calculated. The mentioned coefficients can be found by machine tests e.g. a locked rotor test, which leads to invest more time to progress it.
For that reason, the nonlinear BH curve for the inductance and the neutral point have to be added into the model. Due to that, the mutual inductances between phases or the coupled flux of other phases as shown in the DFC calculation cannot be neglected. Therefore, it leads to the difficulty to model the machine to react as the real PMSM in Simulink.

A solution to overcome the mentioned obstacles is to use a finite element method (FEM) as in [28 – 30]. A cooperative simulation with Simplorer and Maxwell is selected. Maxwell can compute many kinds of calculations by FEM and many values e.g. the flux density and the inductance, which cannot be measured and extracted in other simulation environments. Even signals which are difficult to obtain from real machines can be attained. The nonlinear BH curve can be included into the machine model. Nevertheless, all machine data and properties, i.e. defining coil terminals, group coil terminals to phase windings, the permanent magnets properties, allocation of material properties and magnets including the BH curve of the lamination stack, the rotor and the stator lamination stack geometry, and assign the excitation direction, are required. Otherwise, the machine model cannot behave as the real machine.

For Simplorer, the co-simulation with Maxwell is available and many relevant parts, i.e. the drive system, the measuring system, the modulation algorithm, and extra programs in C language, can be added.

The simulation environment of the complete combination between Simplorer and Maxwell from Simplorer is shown in Fig. 3.11.

From Fig. 3.11, the DFC method with the modulation algorithm is programmed in C by the built in C editor and the PMSM model is linked to the PMSM modeled in Maxwell. This environment is firstly investigated in [31]. There are two inputs, i.e. a DFC input and a load. The DFC input is a maximum duty cycle of the PWM unit in percentage, which is used to generate the voltage vector. The load is another machine, which is connected to the PMSM model shaft. The load can be applied by assigning a constant speed.
According to the machine models, all three machines, i.e. PMSM1, PMSM2, and PMSM3, 2 dimensional (2D) models have been achieved in Maxwell as illustrated in Fig. 3.12. However, only data and all properties of PMSM2 are available. Therefore, the PMSM2 model is validated and experimented. It is worth to mention that 3D models can be also implemented in Maxwell. Due to the high computational complexity of the 3D model, the 2D model is selected.
After modeled and setup PMSM2, the model has to be validated to confirm its behaviors to be similar to the real PMSM2. Therefore, a no load running test is applied to examine both the real PMSM2 and the modeled PMSM2. Both are driven at 214 rounds per minute (rpm) and line to line induced voltages ($V_{UV}$) are measured, which are depicted in Fig. 3.13. The $V_{UV}$ signals have the same contents, i.e. frequency, amplitude, and shape, except a little distortion on the FEM model signal. This is because the simulation sampling time is set to the maximum possible value (1 μs) to optimize the computational complexity. Hence, the PMSM2 model has the correct characteristics as the real PMSM2.

![Image](image.png)

**Fig. 3.13:** FEM model validation

Subsequently, there are two experiments, which are tested and investigated with the PMSM2 model. Firstly, 20% (the maximum duty cycle of the PWM unit, $P_d$) is applied as the DFC input and the experiment time duration is 60 ms. The experimental results are the flux linkage signals ($u,v,w$) and the calculated electrical rotor position ($\alpha_{cal}$) as depicted in Fig. 3.14.

Next, the PMSM2 model is driven by the load machine as the step input at 35 rpm, the experiment is performed for 35 ms. The DFC input is set to a small value e.g. 4% , because $V_{NAN}$ is required to measure and the applied input must be the short duty cycle which does not have any influence to drive the PMSM. The flux linkage signals have been extracted and measured as shown in Fig. 3.15(a). $\alpha_{cal}$ is compared with the ideal electrical rotor position ($\alpha$), which is represented in Fig. 3.15(b).
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Fig. 3.14: Applied constant input at $P_M = 20\%$

a) Flux linkage signals ($u$: red, $v$: blue, $w$: green)

b) Calculated electrical rotor position ($\alpha_{cal}$)
a) Flux linkage signals (u: red, v: blue, w: green)

b) Comparison of calculated electrical positions (α_cal: blue, α: red)

Fig. 3.15: Applied a constant speed to load machine at 35 rpm
According to the experimental results, the flux linkage signals can be extracted in both experiments. They are periodical signals with similar shape and each signal is phase shifted as three phase system. $\alpha_{cal}$ can be also found. A period of $\alpha_{cal}$ is from 0 to 360 degrees, which means that two periods of flux linkage signals are used. In this case, the PMSM2 has 40 rotor magnet poles. Hence, there are 40 periods of each phase flux linkage signal and 20 periods of calculated electrical rotor position for one mechanical revolution. The ideal electrical rotor position ($\alpha$) and $\alpha_{cal}$ are similar to each other as in Fig. 3.15(b), except in the beginning. This is because $\alpha_{cal}$ is related to influence of the machine stator currents, which is not the same as the ideal position. $\alpha_{cal}$ is from 0 to 145 degrees, approximately. It conforms to the period of the flux linkage signals, which is less than a period. It shows that the machine information can be found by DFC much more than with the mechanical sensors.

### 3.2.2 Hardware Implementation

All PMSMs are implemented on a real time controller based on a TriCore PXROS platform. The microcontroller is a TC1796 (32-bit TriCore™ Microcontrollers). The TCP/IP protocol is a communication protocol between the TriCore PXROS platform and a host computer. Each machine experimental setup and results are described as following.

#### 3.2.2.1 PMSM1

PMSM1 is connected to the TriCore PXROS platform, where the driving circuit, an artificial neutral point circuit, and a very fast analog to digital converter (FADC) with 10 bits resolution and 280 ns conversion time are compacted as illustrated in Fig. 3.16. $V_{DC}$ for PMSM1 is 12 V and the frequency of the PWM unit is 20 kHz. The motor is driven by a MOSFET power stage.

Moreover, the test bench for small machines is also designed in order to test other motors as shown in Fig. 3.17. The tested machine shaft is coupled to a load motor shaft. Both motors can be driven by using the host computer including collecting the experiment data at the same time.
Fig. 3.16: Connected PMSM1 with TriCore PXROS platform

Fig. 3.17: Test bench for small machines

Fig. 3.18: Step input at $P_M = 10\%$
Two experiments are achieved. Firstly, a step input in Fig. 3.18 is applied as the DFC input, where $P_M$ is set at 10%. The time duration of the experiment is 180 ms. The flux linkage signals ($u, v, w$) and the calculated electrical rotor position ($\alpha_{cal}$) are the experimental results, which are displayed in Fig. 3.19 (a) and (b), respectively.

Fig. 3.19: Applied step input experimental results
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The experimental results in Fig. 3.19 show that the flux linkage signals are changed, when the input is being changed. Furthermore, the flux linkage signals are also periodical signals as in the software implementation. \( \alpha_{\text{cal}} \) can be attained including standstill from 0 to 50 ms.

Next, the varied inputs, i.e. 10\%, 0, and -10\% are applied to drive PMSM1 as shown in Fig. 3.20. Each level is changed every 112.5 ms, which is a very small period. The main purpose of the experiment is to assure that the DFC method can obtain \( \alpha_{\text{cal}} \) for all speeds, i.e. clockwise and anticlockwise directions, and at standstill. The results are plotted in Fig. 3.21.

![Varied input pattern](image)

Fig. 3.20: Varied input pattern

Fig. 3.21 shows that the electrical rotor positions are obtained for all speeds and at standstill. The delay occurs whilst the direction is changed, because of the machine mechanical time constant. At standstill, the calculated electrical rotor position slightly swings. This is because the flux linkage signals oscillate, which can be caused by the effect of the moment of inertia of the motor due to a very short time experiment and the calculated position can be at the middle of a rotor magnetic pole. The middle of a rotor magnetic pole strongly influences the variance of the inductance and directly interacts with the stator teeth, where the influence of stator currents is available, which can generate the cogging torque. To compare the calculated electrical rotor
position in Fig. 3.19 (b) and 3.21 (b), the directions are not the same while being applied the positive DFC input. It means that the uncertainty of rotor poles occurs in this case.

a) Flux linkage signals (u: red, v: blue, w: green)

b) Calculated electrical rotor position ($\alpha_{cal}$)

Fig. 3.21: Applied varied input experimental results
3.2.2.2 PMSM2

PMSM2 is also connected to the TriCore PXROS platform, but the driving circuit is not compacted with the system. A different inverter stack in the laboratory with IGBTs is selected to use. This is because the compacted system can deal only with two levels of the DC link voltage, i.e. 12 and 24 V. Therefore, a system with a wide range of the DC link voltage \(V_{DC}\) is needed. The interface unit to use for the wide range of the DC link voltage has been built and combined together with the TriCore PXROS platform in [32], which is called test bench for big machines as shown in Fig. 3.22. The voltage divider circuits and the artificial point circuits, which are suitable for higher DC link voltage levels i.e. 60, 200, and 400 V, are also designed.

![Fig. 3.22: Test bench for big machines](image)

PMSM2 is experimented in the same way as PMSM1. The PWM frequency is 10 kHz. 24 V is a required voltage level for PMSM2, but the DC link voltage is set to 30 V. This is because some voltages are dropped in the IGBT circuit. The applied varied input pattern is shown in Fig. 3.23. The experimental results are depicted in Fig. 3.24, which are similar to the PMSM1 experimental results in Fig. 3.21.

It is noteworthy that all PMSM1 and PMSM2 experiments are done without any control strategies. In order to improve the mentioned characteristics, a control strategy to work with DFC is necessary.
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**Fig. 3.23:** Varied input pattern

a) Flux linkage signals ($u$: red, $v$: blue, $w$: green)
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b) Calculated electrical rotor position ($\alpha_{cal}$)

Fig. 3.24: Applied varied input experimental results

3.2.2.3 PMSM 3

PMSM3 is implemented on the same system as with PMSM2. The DC link voltage of PMSM3 is 200 V. Three levels of input ($P_m$), 27%, 0, and -27%, are applied to drive PMSM3. Each level is slightly changed by a ramp function. The experiment time duration is 2400 ms. The flux linkage signals are illustrated in Fig. 3.25.

The PMSM3 results cannot be accurately used to calculate the correct electrical rotor positions. This is because the PMSM3 structure is different from PMSM1 and PMSM2. The rotor magnets are buried, which leads to have less variance of inductance or machine saliencies. Thus, PMSM3 is called non-salient poles PMSM, which has a special design to generate smooth torque by keeping fluxes constant in the air gap.

However, better flux linkage signals can be obtained by modifying the DFC input. This is because the input can generate the stator fluxes, which can directly effect to the machine saliencies, for instance the working point is moved to the saturated area of the magnetization curve. An electrical correction angle ($\alpha_k$) or the commutation
advance angle is introduced to improve the machine saliencies. $\alpha_k$ can be combined with the three phase input voltage as in (3.21), where $P_M$ is the original input of the DFC. The DFC diagram to implement with all PMSMs is represented in Fig. 3.26.

\[
V_U = \frac{P_M}{100} V_{DC} \sin(\alpha_{cal} + \alpha_k)
\]
\[
V_V = \frac{P_M}{100} V_{DC} \sin(\alpha_{cal} + \alpha_k + \frac{2\pi}{3})
\]
\[
V_W = \frac{P_M}{100} V_{DC} \sin(\alpha_{cal} + \alpha_k - \frac{2\pi}{3})
\]

Fig. 3.25: PMSM3 flux linkage signals (u: red, v: blue, w: green)

Fig. 3.26: DFC Diagram
Besides, the experiment to find the proper $\alpha_k$ values can be done by varying $P_M$ and adjusting $\alpha$, manually. The proper $\alpha_k$ is selected by considering on the flux linkage signal characteristics. They must be less noisy and sufficient enough to calculate the electrical rotor positions. It means that the inputs, i.e. the stator currents, generate a field component, which affects through the operating point of the magnetic circuit e.g. magnetic field strengthening in the direct axis of the synchronous frame. The example experiment to get the proper $\alpha_k$ values for PMSM3 has been done in [32].

After experimenting, it shows that the relation between $\alpha_k$ and $P_M$ is nonlinear. Consequently, PMSM3 is possible to work with DFC by using the found relation. There are two possibilities to use the found relation, i.e. direct e.g. lookup tables and indirect e.g. artificial intelligence. Fuzzy logic is one of the well known methods, which is used to overcome the problems in many fields, and also in nonlinear systems e.g. electrical machines control problems [33] – [35]. Moreover, it is better than using look up tables, this is because some of the desired values are overlapped with each other. For instance, the flux intensity management (FIM) by fuzzy logic is proposed to drive PMSM3 with DFC and to compensate the asymmetrical system in [36].
4 Analysis of Direct Flux Control

After performing several experiments in the previous chapter, there are several aspects to consider, which are explained respectively.

4.1 DFC Implementation Restriction

In order to implement DFC, there are several restrictions, which have to be taken into account. They can be listed as follows:

- The DFC method can only deal with the machines, whose neutral points are accessible.

- A very fast analog to digital converter (FACD) to measure $V_{NAN}$ is necessary for DFC in order to run the PMSMs in a wide speed range and to be practicable to work synchronized with the PWM unit.

- As mentioned, the pulses must not be turned on at the same time and a small delay is needed between the switched pulses to acquire the assessable $V_{NAN}$. The assessable $V_{NAN}$ means that the level of $V_{NAN}$ is at steady state. Consequently, an optimal time has to be found. Regularly between two applied pulses, there are two time constant values, i.e. measuring time after switching on pulse ($t_m$) and switching on time of the next pulse ($t_o$). $t_m$ is always less than $t_o$, which can be explicitly explained in Fig. 4.1.

![DFC timing diagram](image)

**Fig. 4.1:** DFC timing diagram
In Fig. 4.1, there are four states of $t_{m}$, which conforms to four measuring states in Chapter 3. $t_{m0}$ is the measuring time before switching on the first pulse, where the steady state time is not needed to consider. $t_{m1}$, $t_{m2}$, and $t_{m3}$ are and the measuring time after switching on from the first to the third pulse and equal to $t_{m}$. For instance, the time constant values of PMSM1, i.e. $t_{m0}$, $t_{m}$, and $t_{o}$ are 200 ns, 800 ns, and 1000 ns, respectively. These values are related to the motor time constant, which can be found by using the time to reach the steady state of the switched phase current. Although the times for switching on the pulses are fixed, any modulation technique can be applied. This is because the desired voltage vector is not changed by using the same calculated voltage time area (VTA) of each pulse.

### 4.2 Flux Linkage Signal Characteristics

The flux linkage signals are the most important signals for DFC and they are utilized to calculate the electrical rotor position. Moreover, they can be obtained for all speeds and at standstill. However, the experiments in Chapter 3 have been done without applying any load. Therefore, the experiments with applied load as a disturbance are investigated. PMSM1 and PMSM2 are driven by applying $P_{M}$ at 10% and 20%, respectively. The applied disturbance is to lock the rotor by hand and this has been applied for a short time to PMSMs while running. The PMSM1 and PMSM2 rotors are locked for 630 ms and 500 ms, approximately.

![Flux linkage signals](image)

a) Flux linkage signals ($u$: red, $v$: blue, $w$: green)
b) Calculated electrical rotor position ($\alpha_{cal}$)

Fig. 4.2: Applied Disturbance on PMSM1

Each machine experimental results are depicted in Fig. 4.2 and 4.3, accordingly. The results show that the calculated electrical rotor positions are constant while applying the disturbance and commonly change after removing the disturbance. It means that the DFC method can deal with the disturbance and the calculated electrical rotor position can be found, even though the flux linkage signal contains spikes. The spikes are caused by the FADC least significant bit and the noises e.g. electromagnetic interferences (EMI) in the real time implementation.

a) Flux linkage signals ($u$: red, $v$: blue, $w$: green)
In addition, the extracted electrical rotor signal is usually assumed as a periodical signal and the electrical rotor position can be obtained by using signal processing methods based on the high frequency machine equation [11 – 24]. Thus, the flux linkage signal of PMSM2 is captured as shown in Fig. 4.4(a) and the signal is normalized and transformed into the frequency domain. The normalized spectrum of the flux linkage signal is plotted and zoomed in Fig. 4.4(b) and 4.4(c).

---

**Fig. 4.3: Applied Disturbance on PMSM2**

---

**b) Calculated electrical rotor position \( \alpha_{\text{cal}} \)**

---

**a) Captured flux linkage signal**
Regarding the dominant frequencies, there are three frequencies, i.e. DC component, the second, and the fourth harmonics, the normalized flux linkage signal approximation function can be stated in (4.1).

\[
\hat{u} = 0.21 \cos(2 \omega_p t) + 0.17 \cos(4 \omega_p t)
\]  

(4.1)
Where \( \hat{u} \) is the approximated flux linkage signal. \( \omega_p \) is the electrical angular frequency and \( t \) is the time. Based on the relation in (4.1), it is used to estimate the flux linkage signal as depicted in Fig. 4.5. \( \omega_p \) is set to 141.37 rad/s and a signal gain of 110 is applied, approximately.

The approximated flux linkage signal in Fig. 4.5 is almost the same as the captured signal in Fig. 4.4 (a). It is worth to mention that the high frequency model [11 – 24] is based on the second harmonic to extract the electrical position, which is the inductance frequency. The second harmonic is also available in the DFC flux linkage signal.

![Fig. 4.5: Approximated flux linkage signal](image)

4.3 Influence of Stator Currents

The DFC calculation is directly related to the resultant flux linkage \( (\Psi_p) \) and finalized in the short form as \( L_{\sigma p} \). The relation is rewritten in (4.2).

\[
L_{\sigma p} = L_{\sigma p} + I_p \frac{dL_{\sigma p}}{dI_p}
\]  \hspace{1cm} (4.2)
As represented in (4.2) and the several experimental results e.g. Fig. 3.21, 3.25, and 4.3, the generated stator flux ($\Psi_s$) by the stator currents can influence the flux linkage signal in some cases. Thus, $\Psi_s$ is necessary to decouple from $\Psi_p$ in order to obtain the exact electrical rotor position, which is required for the control strategies. The relation between $\Psi_s$ and $\Psi_p$ is in (4.3) and represented in Fig. 4.6. Where $\Psi_r$ is the rotor flux.

$$\Psi_p = \Psi_s + \Psi_r$$  \hspace{1cm} (4.3)

Fig. 4.6: Flux relations in stationary frame

### 4.4 Influence of Different PMSM Structures on DFC

The accessibility of the machine neutral point is compulsory for the PMSM structure to implement DFC. Moreover, the experimental results between the salient-poles PMSM (PMSM1 and PMSM2) and the non-salient-poles PMSM (PMSM3) are basically different based on the structure of the machine rotor, which lead to the magnetic field characteristics. In fact, there are many factors of the PMSM structure, e.g. inductances, core losses, cogging torque, pair of poles, magnet geometry, as briefed in [37] which have to be considered.

Consequently, the existing validated PMSM2 model in software simulation environment is selected to do further investigations as in [38]. PMSM2 has 54 stator teeth, 20 rotor pole pairs including a blank space from removed Hall sensor as shown in Fig. 3.9. The modification has been done by modifying the number of stator teeth.
and also balancing the blank space. The number of rotor pole pairs remains the same. There are four modified stator numbers, i.e. 30, 45, 54 and 57 slots. Each modified PMSM2 model is experimented by driving the load machine at 35 rpm as same as the second experiment in software implementation.

After examining, the calculated electrical rotor position of each modified model is depicted in Fig. 4.7. Each result is compared to the ideal electrical position, the differences are plotted in Fig. 4.8. Each modified model $V_{NAN}$ and machine torque ($T_m$) are represented in Fig. 4.9 and 4.10, respectively.

Fig. 4.7 shows that $\alpha_{cal}$ for all modified PMSMs is close to the ideal electrical rotor position, except the 30 slots model. Therefore, the 45, 54, and 57 slots models errors are calculated by subtraction with the ideal position and considered. It is found that the error is less when the number of stator slots is increased, which conforms to $V_{NAN}$ and cogging torque ($T_{cog}$) behavior. The 57 slots model $V_{NAN}$ is smoother than the 54 slots model.

Besides, $T_{cog}$ is an important factor to analyze PMSM, it shows the interaction between stator teeth and rotor poles. The $T_{cog}$ cycles can be found using the least common multiple (LCM) of number of stator teeth and number of rotor poles [37]. High $T_{cog}$ cycles are desired because increasing cogging torque frequencies decrease the cogging torque magnitude.

![Graph showing electrical rotor position for different number of slots](image)

Fig. 4.7: Each modified PMSM electrical rotor position
Analysis of Direct Flux Control

Fig. 4.8: Calculated electrical rotor position errors

Fig. 4.9: Each modified PMSM $V_{NAN}$

Fig. 4.10: Each modified PMSM $T_m$
All distinct aspects of structure analysis experimental results from Fig. 4.7 to 4.10 are represented in Table 4.1. The most suitable model PMSM to implement DFC is the 57 slots model. It means that the DFC method works well with the machine, which has an asymmetry structure and high LCM value. This is because these two conditions create better machine saliencies and distribute the inductance inside the machine.
5 Derivation of Direct Flux Control Signals

Based on the evaluated signals to estimate the electrical rotor position in sensorless methods e.g. [11 – 24], the high frequency machine model and the characteristic of the second harmonic of the phase inductances are used.

Regarding the DFC method, the flux linkage signals, i.e. $u$, $v$, and $w$, spectrums consist of three dominant frequencies as elucidated in the previous chapter. Consequently, the mathematical expression of the DFC signals or the flux linkage signals is necessary in order to investigate and develop the DFC method.

The derivation has been done and discussed in several parts as listed below:

- Relation between fluxes in the synchronous frame ($d$, $q$) and the stator frame ($U$, $V$, $W$).
- Apply the flux relation to the machine voltage equation to obtain the equation of the voltage at the neutral point ($V_N$).
- Apply the DFC conditions including the added artificial neutral point voltage ($V_{AN}$) to attain $V_{NAN}$ at each step.
- Conclude the flux linkage signals equation for all speeds and at standstill.
- Removing the uncertainty of the calculated position.

The theoretical and practical approaches are explained in parallel in each part.

5.1 Fluxes in Stator Frame

As in the calculation of Direct Flux Control is done by using $V_{NAN}$, the fluxes equations in the stator frame have to be known.

Basically, the values in ($d$, $q$) frame are used to reference the calculation values. There are several ways to setup the reference frames and calculate as in [37], [39 – 41]. In this chapter, the frame is based on the space vector diagram in Fig. 5.1, which is implemented on the TriCore PXROS platform. All flux leakages, which are generated by leakage inductances in ($d$, $q$), are not taken into account. The fluxes in $d$ and $q$ axis are $\Psi_d$ and $\Psi_q$, respectively. $\Psi_r$ is the rotor flux, which is from the permanent magnet and always aligns on the $d$ axis. Thus, $\Psi_d$ and $\Psi_d$ can be represented as in (5.1).
Derivation of Direct Flux Control Signals

**Fig. 5.1:** Space vector diagram on TriCore PXROS platform

\[ \Psi_d = L_d I_d + \Psi_r \]
\[ \Psi_q = L_q I_q \]  

(5.1)

Where \( L_d \) and \( L_q \) are the inductances in \( d \) and \( q \) axis, \( I_d \) and \( I_q \) are the currents, which are generated by the phase currents, i.e. \( I_U \), \( I_V \), and \( I_W \).

In order to obtain the total phase fluxes, i.e. \( \Psi_{t,U} \), \( \Psi_{t,V} \), and \( \Psi_{t,W} \) as in (5.2), \( \Psi_r \) of each phase can be calculated as in (5.3). Moreover, \( \Psi_s \) of each phase can be found by converting \( \Psi_d \) and \( \Psi_q \) to the stator frame, which can be done in four steps as following:

\[ \Psi_{t,U} = \Psi_{s,U} + \Psi_{r,U} \]
\[ \Psi_{t,V} = \Psi_{s,V} + \Psi_{r,V} \]
\[ \Psi_{t,W} = \Psi_{s,W} + \Psi_{r,W} \]  

(5.2)

\[ \Psi_{r,U} = \Psi_r \cos(\alpha) \]
\[ \Psi_{r,V} = \Psi_r \cos(\alpha + \frac{2\pi}{3}) \]
\[ \Psi_{r,W} = \Psi_r \cos(\alpha + \frac{4\pi}{3}) \]  

(5.3)
5.1.1 Convert Phase Currents to (d,q) Frame

In the TriCore PXROS platform the phase voltage equation is defined as in (5.4), which can be used to assume the phase currents at standstill and without load to be the same as in (5.5), where \( V \) and \( I \) are the magnitude of phase voltage and phase current.

\[
V_{u} = V \sin(\alpha) \\
V_{v} = V \sin(\alpha + \frac{2\pi}{3}) \tag{5.4} \\
V_{w} = V \sin(\alpha + \frac{4\pi}{3}) \\
I_{u} = I \sin(\alpha) \\
I_{v} = I \sin(\alpha + \frac{2\pi}{3}) \tag{5.5} \\
I_{w} = I \sin(\alpha + \frac{4\pi}{3})
\]

Normally, the three phase currents can be converted into the stationary frame \((\alpha, \beta)\) by using \( \frac{\sqrt{3}}{2} \) as the gain with the symmetry system, the power invariant conversion and other electrical values e.g. voltages, for the fluxes the same matrix to do the conversion as in (5.6) and rearranged in (5.7) can be used.

\[
\begin{bmatrix} \sqrt{3} I_\alpha \\ \sqrt{2} I_\beta \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{3} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} I_u \\ I_v \\ I_w \end{bmatrix} \tag{5.6}
\]

\[
\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} I_u \\ I_v \\ I_w \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{6}} (-\sqrt{3}I_v + \sqrt{3}I_w) \\ \frac{1}{\sqrt{6}} (2I_v - I_v - I_w) \end{bmatrix} \tag{5.7}
\]

The currents in the stationary frame can be transformed to the synchronous frame by using the rotation matrix as in (5.8) and (5.9), respectively.

\[
\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin(\alpha) & \cos(\alpha) \\ \cos(\alpha) & -\sin(\alpha) \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} \tag{5.8}
\]
Derivation of Direct Flux Control Signals

\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} = \frac{1}{\sqrt{6}} \left[ (-\sqrt{3}I_V + \sqrt{3}I_W)\sin(\alpha) + (2I_U - I_V - I_W)\cos(\alpha) \right]
\]
\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} = \frac{1}{\sqrt{6}} \left[ (-\sqrt{3}I_V + \sqrt{3}I_W)\cos(\alpha) - (2I_U - I_V - I_W)\sin(\alpha) \right]
\]

\[(5.9)\]

5.1.2 Fluxes in (d,q) Frame Calculation

As stated in (5.1) to (5.3), \(\Psi_d\) is the combination between \(\Psi_r\) and \(L_dI_d\), \(\Psi_q\) is \(L_qI_q\). \(\Psi_r\) can be separated from the calculation and superposed in the end of the calculation. Thus, the terms \(L_dI_d\) and \(L_qI_q\), i.e. \(\Psi_{s,d}\) and \(\Psi_{s,q}\), have to be considered. \(\Psi_{s,d}\) and \(\Psi_{s,q}\) can be calculated in (5.10).

\[
\begin{bmatrix}
\Psi_{s,d} \\
\Psi_{s,q}
\end{bmatrix} = \frac{L_d}{\sqrt{6}} \left[ (-\sqrt{3}I_V + \sqrt{3}I_W)\sin(\alpha) + (2I_U - I_V - I_W)\cos(\alpha) \right]
\]
\[
\begin{bmatrix}
\Psi_{s,d} \\
\Psi_{s,q}
\end{bmatrix} = \frac{L_q}{\sqrt{6}} \left[ (-\sqrt{3}I_V + \sqrt{3}I_W)\cos(\alpha) - (2I_U - I_V - I_W)\sin(\alpha) \right]
\]

\[(5.10)\]

5.1.3 Convert Fluxes in (d,q) Frame to Stator Frame

\(\Psi_{s,d}\) and \(\Psi_{s,q}\) can be converted to \(\Psi_{s,U}\), \(\Psi_{s,V}\) and \(\Psi_{s,W}\) by using (5.11) and (5.12), respectively. The calculation in this part is described in appendix 10.1.

\[
\begin{bmatrix}
\Psi_{s,\alpha} \\
\Psi_{s,\beta}
\end{bmatrix} = \begin{bmatrix}
\sin(\alpha) & \cos(\alpha) \\
\cos(\alpha) & -\sin(\alpha)
\end{bmatrix} \begin{bmatrix}
\Psi_{s,d} \\
\Psi_{s,q}
\end{bmatrix}
\]

\[(5.11)\]

\[
\begin{bmatrix}
\Psi_{s,U} \\
\Psi_{s,V} \\
\Psi_{s,W}
\end{bmatrix} = \begin{bmatrix}
0 & \sqrt{2}/3 \\
1/\sqrt{2} & -1/\sqrt{6} \\
1/\sqrt{2} & 1/\sqrt{6}
\end{bmatrix} \begin{bmatrix}
\Psi_{s,\alpha} \\
\Psi_{s,\beta}
\end{bmatrix}
\]

\[(5.12)\]

5.1.4 Fluxes in Stator Frame Calculation

As results, \(\Psi_{s,U}\), \(\Psi_{s,V}\) and \(\Psi_{s,W}\) can be given in (5.13) and (5.14), which are used to calculate the phase total fluxes as rewritten in (5.15).
Derivation of Direct Flux Control Signals

\[
\begin{bmatrix}
\Psi_{s,U} \\
\Psi_{s,V} \\
\Psi_{s,W}
\end{bmatrix} =
\begin{bmatrix}
I_U & I_{UV} & I_{UW} \\
I_{VU} & I_V & I_{VW} \\
I_{WU} & I_{WV} & I_w
\end{bmatrix}
\]

\[(5.13)\]

\[
L_U = \frac{1}{3}((L_d + L_q) + (L_d - L_q) \cos(2\alpha))
\]

\[
L_{UV} = \frac{1}{3}(\frac{1}{2}(L_d + L_q) + (L_d - L_q) \cos(2\alpha + \frac{2\pi}{3}))
\]

\[
L_{UW} = \frac{1}{3}(\frac{1}{2}(L_d + L_q) + (L_d - L_q) \cos(2\alpha + \frac{4\pi}{3}))
\]

\[
L_{VU} = \frac{1}{3}(\frac{1}{2}(L_d + L_q) + (L_d - L_q) \cos(2\alpha + \frac{2\pi}{3}))
\]

\[
L_V = \frac{1}{3}((L_d + L_q) + (L_d - L_q) \cos(2\alpha + \frac{4\pi}{3}))
\]

\[
L_{VW} = \frac{1}{3}(\frac{1}{2}(L_d + L_q) + (L_d - L_q) \cos(2\alpha))
\]

\[
L_{WU} = \frac{1}{3}(\frac{1}{2}(L_d + L_q) + (L_d - L_q) \cos(2\alpha + \frac{4\pi}{3}))
\]

\[
L_{WV} = \frac{1}{3}(\frac{1}{2}(L_d + L_q) + (L_d - L_q) \cos(2\alpha))
\]

\[
L_w = \frac{1}{3}((L_d + L_q) + (L_d - L_q) \cos(2\alpha + \frac{2\pi}{3}))
\]

\[(5.14)\]

\[
\Psi_{s,U} = \Psi_{s,U} + \Psi_{r,U}
\]

\[
\Psi_{s,V} = \Psi_{s,V} + \Psi_{r,V}
\]

\[
\Psi_{s,W} = \Psi_{s,W} + \Psi_{r,W}
\]

\[(5.15)\]

The derived values show that \(L_{UV}, L_{UV}, L_{UW}, L_{VW}, \) and \(L_{WV}\) are the same, which can be called as the mutual inductances. \(L_U, L_V,\) and \(L_W\) are the phase inductances. All calculated inductances angles are two times the angle of the electrical rotor position. Thus, the derived inductances conform to the signal behaviors of sensorless methods based on the machine saliencies and the angle dependencies.
5.2 Voltage Equation at the Machine Neutral Point ($V_N$)

After knowing the fluxes in stator frame as in (5.15), the PMSM stator model can be depicted in Fig. 5.2. The three phase voltage equation can be stated as in (5.16).

$$
\begin{bmatrix}
V_U \\
V_V \\
V_W
\end{bmatrix} =
\begin{bmatrix}
R_U & 0 & 0 \\
0 & R_V & 0 \\
0 & 0 & R_W
\end{bmatrix}
\begin{bmatrix}
I_U \\
I_V \\
I_W
\end{bmatrix} +
\begin{bmatrix}
\frac{d\Psi_{r,U}}{dt} \\
\frac{d\Psi_{r,V}}{dt} \\
\frac{d\Psi_{r,W}}{dt}
\end{bmatrix} + V_N
$$

(5.16)

The machine voltage equation can be rewritten in (5.17) and expanded in (5.18), where the superscript ' is the first derivative by time.

$$
V_U = I_U R_U + \frac{d(L_{uU} I_U)}{dt} + \frac{d(L_{qv} I_V)}{dt} + \frac{d(L_{qW} I_W)}{dt} + \frac{d\Psi_{r,U}}{dt} + V_N
$$

$$
V_V = I_V R_V + \frac{d(L_{qv} I_U)}{dt} + \frac{d(L_{vV} I_V)}{dt} + \frac{d(L_{qW} I_W)}{dt} + \frac{d\Psi_{r,V}}{dt} + V_N
$$

$$
V_W = I_W R_W + \frac{d(L_{qW} I_U)}{dt} + \frac{d(L_{vW} I_V)}{dt} + \frac{d(L_{wW} I_W)}{dt} + \frac{d\Psi_{r,W}}{dt} + V_N
$$

(5.17)
V_u = I_u R_u + L'_u I_u + L'_uv I_v + L'_uv I_v + L'_uv I_v + L'_uv I_v + L'_uv I_v + L'_uv I_v + L'_uv I_v + L'_uv I_v + \Psi'_{r,u} + V_N

V_v = I_v R_v + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + \Psi'_{r,v} + V_N

V_w = I_w R_w + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + L'_uv I_u + \Psi'_{r,w} + V_N

The voltage equations can be rearranged as in (5.19).

\[ I'_u = \frac{(V_u - I_u R_u - L'_u I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - \Psi'_{r,u} - V_N)}{L_u} \]

\[ I'_v = \frac{(V_v - I_v R_v - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - \Psi'_{r,v} - V_N)}{L_v} \]

\[ I'_w = \frac{(V_w - I_w R_w - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - L'_uv I_u - \Psi'_{r,w} - V_N)}{L_w} \]

The summation of either the phase currents or the first derivatives of the phase currents are always zero in a three phase system as in (5.20).

\[ I_u + I_v + I_w = 0 \]
\[ I'_u + I'_v + I'_w = 0 \]

V_N can be found by summing all in (5.19), which is represented in (5.21) and concluded in (5.22).
Derivation of Direct Flux Control Signals

\[
V_N = \frac{V_U + V_V + V_W}{L_U + L_V + L_W} - \frac{I_U R_U + I_V R_V + I_W R_W}{L_U + L_V + L_W}
\]

\[
- \frac{L'_{UV} I_U + L'_{UV} I'_V + L'_{UV} I'_V + L'_{UV} I'_W + L_{UV} I'_w}{L_U} \]

\[
- \frac{L'_{UV} I_U + L'_{UV} I'_V + L'_{UV} I'_V + L'_{UV} I'_W + L_{UV} I'_w}{L_U + L_V + L_W}
\]

\[
- \frac{L'_{UV} I_U + L'_{UV} I'_V + L'_{UV} I'_V + L'_{UV} I'_W + L_{UV} I'_w}{L_U + L_V + L_W}
\]

\[
- \frac{\Psi'_{r,U}}{L_U} - \frac{\Psi'_{r,V}}{L_V} - \frac{\Psi'_{r,W}}{L_W}
\]

\[
- \frac{1}{L_U + L_V + L_W}
\]

(5.22)

5.3 Direct Flux Control Method Conditions

As described in Chapter 3, the DFC method uses flux linkage signals or DFC signals as voltage signal to estimate the electrical rotor position. The different voltage \(V_{NAN}\) between the machine neutral point \(V_N\) and the artificial neutral point \(V_{AN}\) is used to calculate the flux linkage signals. In this implementation, \(V_{NAN}\) is defined in Fig. 3.1, which is redisplayed in Fig. 5.3 and calculated in (5.23).

![Fig. 5.3: Flux linkage extraction measuring scheme](image-url)
Derivation of Direct Flux Control Signals

\[ V_{NAN} = V_N - V_{AN} \]  \hfill (5.23)

It is worth to mention that the artificial neutral point circuit has been used in several sensorless methods, e.g. [17], [42 – 47]. The main purpose to add it is to obtain the third harmonic of the rotor flux in order to estimate the rotor position. The measured values are obtained only one time in one modulation period, when all pulses are switched on and the phase currents are constant. Then, \( V_{NAN} \) is the zero sequence voltage (ZSV), which depends on the rotor flux and phase inductances variation as in (5.24). Thus, it cannot work at standstill and low speeds.

\[
V_{ZSV} = -\frac{\Psi_{r,U} + \Psi_{r,V} + \Psi_{r,W}}{L_U + \frac{1}{L_V} + \frac{1}{L_W}} \approx -\sin(3\alpha)
\]  \hfill (5.24)

The DFC method can work for all speeds and it is a continuous excitation method by modifying the pulses patterns. The pulses must not be switched on at the same time and \( V_{NAN} \) between states have to be subtracted. The subtracted sequences are based on the sector of the voltage space vector. For instance, the space vector is generated by switching on \( PWM_U \), \( PWM_V \), and \( PWM_W \), respectively. \( V_{NAN} \) has obtained four steps. \( V_N \) and \( V_{AN} \) equivalent circuits are represented in Fig. 5.4 and 5.5. The DC link voltage (\( V_{DC} \)) has two levels, \(+V_{DC}\) and \(0\) V. The flux linkage signals can be calculated in (5.25).

\[
\begin{align*}
    u &= V_{NAN1} - V_{NAN0} \\
    v &= V_{NAN2} - V_{NAN1} \\
    w &= V_{NAN3} - V_{NAN2}
\end{align*}
\]  \hfill (5.25)

![Fig. 5.4: Four states of \( V_N \)](image)
Derivation of Direct Flux Control Signals

\[ V_N = \frac{V_U + V_V + V_W}{L_U + L_V + L_W} - \frac{I_U R_U + I_V R_V + I_W R_W}{L_U + L_V + L_W} \]

\[ = \frac{I_U I_U' + I_U' I_V + I_V' I_V + I_V' I_W + I_W' I_W' + L_U I_U + L_V I_V + L_W I_W}{L_U + L_V + L_W} \]

Fig. 5.4 shows the four measuring steps of \( V_N \). \( V_{N1} \) to \( V_{N3} \) are measured after the switched phase current is constant. Each step has been done in a very short time, energy storages are not changed. It means that the first derivative terms of phase currents and phase inductances are unavailable, which means several terms of \( V_N \) in (5.22), which is rewritten in (5.26), can be neglected as in (5.27). Consequently, \( V_{N0} \) to \( V_{N3} \) can be stated in (5.28) to (5.31), respectively.

\[ V_{N0} \text{ to } V_{N4} \text{ and } V_{AN0} \text{ to } V_{AN4} \text{ have to be known in order to calculate } V_{NAN0} \text{ to } V_{NAN4} \text{ and the flux linkage signals.} \]

\[ V_{AN1} \text{ to } V_{AN3} \text{ are measured after the switched phase current is constant. Each step has been done in a very short time, energy storages are not changed. It means that the first derivative terms of phase currents and phase inductances are unavailable, which means several terms of } V_N \text{ in (5.22), which is rewritten in (5.26), can be neglected as in (5.27). Consequently, } V_{N0} \text{ to } V_{N3} \text{ can be stated in (5.28) to (5.31), respectively.} \]
Derivation of Direct Flux Control Signals

\[
V_N = \frac{V_U + V_V + V_W}{\frac{1}{L_U} + \frac{1}{L_V} + \frac{1}{L_W}} - \frac{I_{U0}R_U + I_{V0}R_V + I_{W0}R_W}{\frac{1}{L_U} + \frac{1}{L_V} + \frac{1}{L_W}} - \frac{\Psi_{r,U} + \Psi_{r,V} + \Psi_{r,W}}{L_U + L_V + L_W}
\] (5.27)

\[
V_{N0} = -\frac{I_{U0\delta}R_U + I_{V0\delta}R_V + I_{W0\delta}R_W}{\frac{1}{L_U} + \frac{1}{L_V} + \frac{1}{L_W}} - \frac{\Psi_{r,U} + \Psi_{r,V} + \Psi_{r,W}}{L_U + L_V + L_W}
\] (5.28)

\[
V_{N1} = \frac{V_{DC}}{\frac{L_U}{L_U} + \frac{L_V}{L_V} + \frac{L_W}{L_W}} - \frac{I_{U1R_U} + I_{V1R_V} + I_{W1R_W}}{\frac{1}{L_U} + \frac{1}{L_V} + \frac{1}{L_W}} - \frac{\Psi_{r,U} + \Psi_{r,V} + \Psi_{r,W}}{L_U + L_V + L_W}
\] (5.29)

\[
V_{N2} = \frac{V_{DC} + V_{DC}}{\frac{L_U}{L_U} + \frac{L_V}{L_V} + \frac{L_W}{L_W}} - \frac{I_{U2R_U} + I_{V2R_V} + I_{W2R_W}}{\frac{1}{L_U} + \frac{1}{L_V} + \frac{1}{L_W}} - \frac{\Psi_{r,U} + \Psi_{r,V} + \Psi_{r,W}}{L_U + L_V + L_W}
\] (5.30)

\[
V_{N3} = V_{DC} \frac{I_{U3R_U} + I_{V3R_V} + I_{W3R_W}}{\frac{1}{L_U} + \frac{1}{L_V} + \frac{1}{L_W}} - \frac{\Psi_{r,U} + \Psi_{r,V} + \Psi_{r,W}}{L_U + L_V + L_W}
\] (5.31)

\(V_{AN0}\) to \(V_{AN4}\) in Fig. 5.5 can be calculated in (5.32). \(R_{ANU}, R_{ANV}\) and \(R_{ANW}\) are much bigger than \(R_U, R_V, \) and \(R_W.\) Hence, the currents in the artificial neutral point can be neglected. \(V_{NAN0}\) to \(V_{NAN4}\) can be computed as in (5.33) to (5.36).
\[ V_{AN0} = 0 \]
\[ V_{AN1} = \frac{2}{3} V_{dc} \]
\[ V_{AN2} = \frac{1}{3} V_{dc} \]
\[ V_{AN3} = V_{dc} \]

\[ V_{NAN0} = -\frac{I_{U0}R_U + I_{V0}R_V + I_{W0}R_W}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{\Psi'_{r,U} + \Psi'_{r,V} + \Psi'_{r,W}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} \]

\[ V_{NAN1} = \frac{V_{dc}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{L_U + \frac{1}{L_U} + \frac{1}{L_W}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{\Psi'_{r,U} + \Psi'_{r,V} + \Psi'_{r,W}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{2}{3} V_{dc} \]

\[ V_{NAN2} = \frac{V_{dc} + V_{dc}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{L_U + \frac{1}{L_U} + \frac{1}{L_W}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{\Psi'_{r,U} + \Psi'_{r,V} + \Psi'_{r,W}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{1}{3} V_{dc} \]

\[ V_{NAN3} = V_{dc} - \frac{I_{U1}R_U + I_{V1}R_V + I_{W1}R_W}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{\Psi'_{r,U} + \Psi'_{r,V} + \Psi'_{r,W}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - V_{dc} \]

After obtaining all \( V_{NAN} \), the flux linkage signals can be extracted as in (5.37) and shown in (5.38) to (5.40).

\[ u = V_{NAN1} - V_{NAN0} \]
\[ v = V_{NAN2} - V_{NAN1} \]
\[ w = V_{NAN3} - V_{NAN2} \]

\[ u = \frac{V_{dc}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{V_{dc} + \frac{V_{dc}}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} + \frac{(I_{U1} - I_{U0})R_U + (I_{V1} - I_{V0})R_V + (I_{W1} - I_{W0})R_W}{L_U + \frac{1}{L_U} + \frac{1}{L_W}} - \frac{2}{3} V_{dc} \]
Derivation of Direct Flux Control Signals

\[
v = \frac{V_{DC}}{L_v} + \frac{(I_{U2} - I_{U1})R_u}{L_u} + \frac{(I_{V2} - I_{V1})R_v}{L_v} + \frac{(I_{W2} - I_{W1})R_w}{L_w} + \frac{1}{3}V_{DC} \quad (5.39)
\]

\[
w = \frac{V_{DC}}{L_w} - \frac{(I_{U3} - I_{U2})R_u}{L_u} - \frac{(I_{V3} - I_{V2})R_v}{L_v} - \frac{(I_{W3} - I_{W2})R_w}{L_w} - \frac{2}{3}V_{DC} \quad (5.40)
\]

Based on one of the DFC conditions, the energy storages are not changed, which leads to have the terms of the phase current difference between states as zero. The flux linkage signals can be found in (5.41), which is almost similar to the DFC principle equation as in (3.17), rewritten in (5.42).

\[
u = \frac{V_{DC}}{L_u} - \frac{2}{3}V_{DC} \quad (5.41)
\]

\[
v = \frac{V_{DC}}{L_v} + \frac{1}{3}V_{DC} \quad (5.41)
\]

\[
w = \frac{V_{DC}}{L_w} - \frac{2}{3}V_{DC} \quad (5.41)
\]

\[
u = \frac{1}{\left(\frac{1}{L_{\sigma_u}} + \frac{1}{L_{\sigma_v}} + \frac{1}{L_{\sigma_w}}\right)}V_{DC} \quad (5.42)
\]

\[
v = \frac{1}{\left(\frac{1}{L_{\sigma_u}} + \frac{1}{L_{\sigma_v}} + \frac{1}{L_{\sigma_w}}\right)}V_{DC} \quad (5.42)
\]

\[
w = \frac{1}{\left(\frac{1}{L_{\sigma_u}} + \frac{1}{L_{\sigma_v}} + \frac{1}{L_{\sigma_w}}\right)}V_{DC} \quad (5.42)
However, all extracted flux linkage signals in (5.41) do not have the same reference level as the principle in (5.42). The different reference levels have to be removed or the flux linkage signals must be in the same reference level.

Consequently, the measuring sequences have been modified in order to obtain all flux linkage signals in the same level. The measuring sequences are changed from four states in one modulation period as in Fig. 5.6 (a) to six states in three modulation periods in Fig. 5.6 (b).

Fig. 5.6: DFC timing diagram
The flux linkage signals of the modified measuring sequence \((u_{\text{raw}}, v_{\text{raw}}, w_{\text{raw}})\) can be attained as in (5.43), and the reference levels are the same as in (5.44).

\[
\begin{align*}
  u_{\text{raw}} &= V_{\text{AN}1,T_1} - V_{\text{AN}0,T_1} \\
  v_{\text{raw}} &= V_{\text{AN}1,T_2} - V_{\text{AN}0,T_2} \quad \text{(5.43)} \\
  w_{\text{raw}} &= V_{\text{AN}1,T_3} - V_{\text{AN}0,T_3}
\end{align*}
\]

\[
\begin{align*}
  u_{\text{raw}} &= \frac{V_{\text{DC}}}{L_u} - \frac{2}{3} V_{\text{DC}} \\
  v_{\text{raw}} &= \frac{V_{\text{DC}}}{L_v} - \frac{2}{3} V_{\text{DC}} \quad \text{(5.44)} \\
  w_{\text{raw}} &= \frac{V_{\text{DC}}}{L_w} - \frac{2}{3} V_{\text{DC}}
\end{align*}
\]

After having the same reference levels, the flux linkage signal can be normalized in (5.46) by using the summation of flux linkage signals in (5.45) in order to remove the offset levels. Then, the summation of the flux linkage signals in (5.46) becomes zero.

\[
\begin{align*}
  uvw &= \frac{u_{\text{raw}} + v_{\text{raw}} + w_{\text{raw}}}{3} = -\frac{1}{3} V_{\text{DC}} \quad \text{(5.45)}
\end{align*}
\]

\[
\begin{align*}
  u &= u_{\text{raw}} - uvw = -\frac{V_{\text{DC}}}{L_u} - \frac{1}{3} V_{\text{DC}} \\
  v &= v_{\text{raw}} - uvw = -\frac{V_{\text{DC}}}{L_v} - \frac{1}{3} V_{\text{DC}} \quad \text{(5.46)} \\
  w &= w_{\text{raw}} - uvw = -\frac{V_{\text{DC}}}{L_w} - \frac{1}{3} V_{\text{DC}}
\end{align*}
\]
5.4 Flux Linkage Signals Behaviors

From the flux model derivation, the stator phase inductances inside the three phase machine depend on the rotor position ($\alpha$) as shown in (5.47).

\[
L_d = \frac{1}{3}((L_d + L_q) + (L_d - L_q)\cos(2\alpha))
\]
\[
L_v = \frac{1}{3}((L_d + L_q) + (L_d - L_q)\cos(2\alpha + \frac{4\pi}{3}))
\]
\[
L_w = \frac{1}{3}((L_d + L_q) + (L_d - L_q)\cos(2\alpha + \frac{2\pi}{3}))
\]

Equation (5.47) can be rewritten in the shorter form in (5.48) with the expression in (5.49).

\[
L_d = L_d + L_q
\]
\[
L_v = L_d - L_q
\]

Consequently, the flux linkage signals or the DFC signals in (5.46) can be found by

\[
u = \frac{(L_d + L_v\cos(2\alpha + \frac{2\pi}{3})) (L_d + L_v\cos(2\alpha + \frac{4\pi}{3}))}{L^2_{sum}} V_{dc} - \frac{1}{3} V_{dc}
\]
\[
v = \frac{(L_d + L_v\cos(2\alpha + \frac{2\pi}{3})) (L_d + L_v\cos(2\alpha))}{L^2_{sum}} V_{dc} - \frac{1}{3} V_{dc}
\]
\[
w = \frac{(L_d + L_v\cos(2\alpha)) (L_d + L_v\cos(2\alpha + \frac{4\pi}{3}))}{L^2_{sum}} V_{dc} - \frac{1}{3} V_{dc}
\]

\[
L^2_{sum} = (L_d + L_v\cos(2\alpha))(L_d + L_v\cos(2\alpha + \frac{2\pi}{3}))
\]
\[
+ (L_d + L_v\cos(2\alpha + \frac{2\pi}{3}))(L_d + L_v\cos(2\alpha + \frac{4\pi}{3})){1\over 3}
\]
\[
+ (L_d + L_v\cos(2\alpha + \frac{4\pi}{3}))(L_d + L_v\cos(2\alpha))
\]
These values, i.e. \( L_2^{\text{sum}} \), \( u \), \( v \) and \( w \) can be calculated in (5.51) and (5.52). The calculation of \( u \), \( v \) and \( w \) is described in appendix 10.2.

\[
L_2^{\text{sum}} = 3L_x^2 - 0.75L_y^2 \tag{5.51}
\]

\[
u = \frac{(L_x^2 - 0.25L_y^2) - L_xL_y \cos(2\alpha) + 0.5L_y^2 \cos(4\alpha)}{3L_x^2 - 0.75L_y^2} V_{DC} - \frac{1}{3} V_{DC} \tag{5.52}
\]

\[
w = \frac{(L_x^2 - 0.25L_y^2) + L_xL_y \cos(2\alpha + \frac{\pi}{3}) + 0.5L_y^2 \cos(4\alpha + \frac{2\pi}{3})}{3L_x^2 - 0.75L_y^2} V_{DC} - \frac{1}{3} V_{DC} \tag{5.54}
\]

Based on the assumption in (5.53), the flux linkage signals in (5.52) can be in (5.54) and finalized as in (5.55).

\[
\xi = \frac{L_x}{L_y} \tag{5.53}
\]

\[
u = \frac{(\xi^2 - 0.25) - \xi \cos(2\alpha) + 0.5 \cos(4\alpha)}{3\xi^2 - 0.75} V_{DC} - \frac{1}{3} V_{DC} \tag{5.54}
\]

\[
w = \frac{(\xi^2 - 0.25) + \xi \cos(2\alpha + \frac{\pi}{3}) + 0.5 \cos(4\alpha + \frac{2\pi}{3})}{3\xi^2 - 0.75} V_{DC} - \frac{1}{3} V_{DC} \tag{5.55}
\]

\[
u = \frac{-\xi \cos(2\alpha) + 0.5 \cos(4\alpha)}{3\xi^2 - 0.75} V_{DC} \tag{5.55}
\]

\[
v = \frac{\xi \cos(2\alpha + \frac{\pi}{3}) + 0.5 \cos(4\alpha + \frac{2\pi}{3})}{3\xi^2 - 0.75} V_{DC} \tag{5.55}
\]

\[
w = \frac{\xi \cos(2\alpha - \frac{\pi}{3}) + 0.5 \cos(4\alpha - \frac{2\pi}{3})}{3\xi^2 - 0.75} V_{DC} \tag{5.55}
\]
The finalized flux linkage signal equations show that each phase has a phase shift of 60 degrees with double frequencies of the fundamental frequency. The DC component is eliminated. However, the DC component can occur anytime, e.g. a small offset in the real time system or the numerical technique in software. Therefore, the finalized equation in (5.55) is similar and conforms to the spectrum analysis in Chapter 4, which are three main spectrums, i.e. a DC component, the second and the fourth harmonics.

As shown in the derivation, the DFC signals are based on two signals, i.e. $V_{DC}$, which is constant and the inductances, which have angle dependencies. Consequently, the DFC signals characteristic are always the same, except when $L_d$ and $L_q$ are extremely changed. Actually, $L_d$ and $L_q$ can be faintly changed as investigated in [48], and in some cases, e.g. at saturation.

Even though $L_d$ and $L_q$ are slightly changed, it does not strongly influence to the flux linkage signals, which are the key of the DFC method. This is because all flux linkage signals have the same structure except the shifted phase. Methods to assure the calculated position are investigated in Chapter 6.

After obtaining the flux linkage signal equations, the flux linkage signals must be depicted and compared with the flux linkage signals of the tested PMSMs.

5.4.1 Flux Linkage Signals from Derived Equation

![Flux Linkage Signals](image)

Fig. 5.7: Calculated flux linkage signals ($u$: red, $v$: blue, $w$: green)
To depict the flux linkage signals, $L_d$ and $L_q$ are assumed to be 0.003 mH and 0.004 mH. $V_{DC}$ is 30 V. One electrical period 0 to 360 degree is taken into account. The flux linkage signals are calculated in MATLAB® and depicted in Fig. 5.7.

### 5.4.2 Flux Linkage Signals of Tested PMSMs

PMSM1 and PMSM2 are considered.

#### 5.4.2.1 PMSM1

The measuring time or time delay ($t_m$) to measure $V_{NAN}$ is set at 800 ns. The phase current of phase $U$ ($I_U$) and the switching pulses are also captured and depicted in Fig. 5.8. $I_U$ remains constant after switched on 800 ns, which means that the first derivative terms can be neglected in the calculation. However, $V_{DC}$ oscillates slightly, which can influence the flux linkage signals.

![Fig. 5.8: Time constant to measure $V_{NAN}$ of PMSM1 ($V_U$: Ch1, $V_V$: Ch2, $V_W$: Ch3, $I_U$: Ch4)](image)

The flux linkage signals of PMSM1 are depicted in Fig. 5.9. The flux linkage signals are extracted without modifying the measuring sequences as described in Fig. 5.6 (a). The sequences of flux linkage signals are also the same as in Fig. 5.7.
Fig. 5.9: PMSM1 flux linkage signals (u: red, v: blue, w: green)

5.4.2.2 PMSM2

The PMSM2 is a bigger motor than the PMSM1. The measuring time \( t_m \) is set at 2700 ns as experimented and depicted in Fig. 5.10. \( I_U \) reaches the steady state before 1 \( \mu \)s. However, \( t_m \) is at 2700 ns in order to have less \( V_{DC} \) oscillation.

Fig. 5.10: Time constant to measure \( V_{XAN} \) of PMSM2 (\( V_U \): Ch1, \( V_V \): Ch2, \( V_W \): Ch3, \( I_U \): Ch4)
The extracted flux linkage signals of PMSM2 on the TriCore PXROS platform are shown in Fig. 5.11. It shows that $u$, $v$, and $w$ of PMSM2 have similar behaviors as in Fig. 5.7, which are without significant offsets and slightly different in magnitudes because of using the modified measured sequences. There are apparent offsets in Fig. 5.9 because of using original measuring sequences.

All flux linkage signals in Fig. 5.7, 5.9 and 5.11 have the same sequences. Thus, the derived equation can be used in order to do further investigations.

The measured sequences are also one of the most important factors in order to design the DFC system. The proper $t_m$ of each machine has to be found. Otherwise, the flux linkage signal is distorted by the first derivative factors and the $V_{DC}$ oscillation. Thus, the DFC method can only use $V_{NAN}$ as the data to process for all speeds and at standstill. Sometimes, the ripple currents after switched on pulses are measured and used to estimate the position as in [49], but it cannot work for all speeds because the first derivative terms are bigger at higher speeds.
5.5 Removing the Uncertainty

The uncertainty of the estimated position is one of the most important problems in sensorless control based on machines saliencies ([11 – 24]). Generally, the main concept is to see the fluxes after applying or injecting the currents, while the magnetic field is either weakening or strengthening as in [11], [12]. Even though the inductance of stator winding at the north pole is less than at the south pole, it cannot be applied for all cases e.g. single tooth winding [50].

Regarding the DFC method, the uncertainty of the calculated electrical position also occurs, which is described in Chapter 3 and shown in Fig. 3.19 (b) and 3.21 (b). Based on the derivation in the previous part, the phase inductances and the flux linkage signals have $L_d$ and $L_q$ as machine parameters in all signals. In regular PMSMs, $L_d$ is always less than $L_q$ ([37], [48]). This is because the rotor magnets are on the $d$ axis, then the reluctance of the magnet is greater in $d$ axis, which means the inductance is less. The inductance is the inverse of the reluctance. Because of saturation, $L_d$ is decreased ($L_{ds}$) , while strengthening the magnetic field. Conversely, $L_d$ is increased ($L_{dw}$), while weakening the magnetic field. The characteristic of $L_d$ and $L_q$ are used to remove the uncertainty.

The flux linkage signals characteristics after changing $L_d$ have to be found. The flux linkage signal characteristics are investigated by setting all parameters as below:

- **Normal Operation:** $L_d$ and $L_q$ are assumed to be 0.003 mH and 0.004 mH. $V_{DC}$ is 30 V.

- **Field Strengthening:** $L_d$ is set to $L_{ds}$, which is equal to 0.0027 mH.

- **Field Weakening:** $L_d$ is set to $L_{dw}$, which is equal to 0.0032 mH.

Each flux linkage signal is depicted in Fig. 5.12. The flux linkage signals of the normal operation are depicted in the first top graph ($u$: red, $v$: blue, $w$: green). The field weakening flux linkage signals ($u_w$, $v_w$, $w_w$) are in red and the field strengthening flux linkage signals ($u_s$, $v_s$, $w_s$) are in green of each graph. They are compared with the normal flux linkage signals ($u$, $v$, $w$), which are in black. The subscript $s$ is for the flux linkage strengthening signal and $w$ is for the flux linkage weakening signal.
The flux linkage weakening and strengthening signals characteristics can be listed as below:

- Positive region: \( u_s > u > u_w \), \( v_s > v > v_w \), \( w_s > w > w_w \)
- Negative region: \( u_s < u < u_w \), \( v_s < v < v_w \), \( w_s < w < w_w \)

Thus, either the highest or the lowest flux linkage signal values can be considered to assure the characteristics, which lead to distinct the position and remove the uncertainty. A removing uncertainty method is explained in the next section.

5.5.1 Removing Uncertainty Methodology

At standstill, the electrical rotor position is estimated by applying a set of pulses pattern of any sector, which does not influence to move the rotor and the machine. The three values \( u, v \) and \( w \) are calculated, which can be used to find the rotor position. However, two rotor positions, i.e. \( \alpha_1 \) and \( \alpha_2 \), are always found based on the calculated flux linkage signals values as in Fig. 5.13. For instance, the calculated flux linkage signal values are \( w > v > u \), then the relation between \( \alpha_1 \) and \( \alpha_2 \) is in (5.56).
Derivation of Direct Flux Control Signals

Fig. 5.13: Flux linkage signals \((u, v, w)\)

Fig. 5.14: Space vector diagram on TriCore PXROS platform
\[ \alpha_2 = \alpha_1 + \pi \] (5.56)

In order to find the exact position, the flux linkage signals have to be considered by recognizing the field weakening flux linkage signals \((u_w, v_w, w_w)\) and the field strengthening flux linkage signals \((u_s, v_s, w_s)\) after applying the current space vectors.

The space vector diagram of the TriCore PXROS platform is depicted in Fig. 5.14. It means that \(u_w, v_w, w_w\) can be obtained when the applied voltage vector \(V_q\), whose phase is similar to \(I_q\) at standstill, to \(d'\) axis, which is called \(I_{q, \alpha_1 + \frac{\pi}{2}}\). For strengthening the field, \(u_s, v_s, w_s\) can be obtained by applying \(I_q\) to \(d\) axis, which is \(I_{q, \alpha_1 + \frac{3\pi}{2}}\). The field weakening and the field strengthening flux linkage signals are used to figure out that \(\alpha_1\) or \(\alpha_2\) is the correct position.

In order to recognize, several steps have to be done as following:

1. Assuming \(\alpha_1\) is the correct position.
2. Applying the current space vector on \(d'\) as \(I_{q, \alpha_1 + \frac{\pi}{2}}\).
3. Increase the magnitude of the space vector. Since the vector aligns on the rotor axis either on the opposite or the same directions, a torque is not created.
4. After increasing the magnitude, the field weakening flux linkage signals \((u_w, v_w, w_w)\) have to be captured.
5. Applying the current space vector on \(d\) as \(I_{q, \alpha_1 + \frac{3\pi}{2}}\).
6. The field strengthening flux linkage signals \((u_s, v_s, w_s)\) must be captured.
7. Comparing the behavior of \(u_w, v_w, w_w\) and \(u_s, v_s, w_s\) based on the regions, where the signals are available. There are two regions, i.e. positive and negative regions, rewritten as following:
   - Positive region: \(u_s > u > u_w, \quad v_s > v > v_w, \quad w_s > w > w_w\)
   - Negative region: \(u_s < u < u_w, \quad v_s < v < v_w, \quad w_s < w < w_w\)
8. If the characteristics are the same as the conditions, \(\alpha_1\) is the correct position. Otherwise, \(\alpha_2\) is the correct position.
In order to assure the performance of the proposed removing uncertainty idea, the experiment has been done with PMSM1. \( u, v, w \) have been captured. The levels are \( u < v < w \), which is the same as the case in Fig. 5.13. Therefore, \( u \) is located in negative region. \( w \) is in positive region. \( v \) is difficult to distinguish between the regions. Thus, \( u \) and \( w \) are only taken into account.

Firstly, \( \alpha_1 \) is assumed to be the correct position. \( I_1, \alpha_1, \frac{\alpha}{2} \) and \( I_1, \alpha_1, \frac{3\alpha}{2} \) are applied, respectively. It is noteworthy that the \( I_q \) magnitude is indirectly increased by applying the maximum duty cycle (\( P_M \)), 2.5% is applied in this experiment. The levels of the flux linkage signals at standstill are \( u < v < w \). If \( \alpha_1 \) is the correct position, \( w \) must be less than \( w_s \) and \( u \) must be greater than \( u_s \) as in Fig. 5.15. Otherwise, \( \alpha_2 \) is the correct position as in Fig. 5.16. Actually, only one flux linkage signal is sufficient enough to consider, e.g. the distinct one in the positive region.

Fig. 5.15: \( \alpha_1 \) correct position
Fig. 5.16: $\alpha_i$ incorrect position

However, the characteristics of the flux linkage signals of each machine have to be investigated firstly. The existing offsets between flux linkage signals can lead to have failures to distinct the calculated position. Both negative and positive regions characteristics have to be considered in order to distinguish the signals with offsets.

After removing the uncertainty and assuring the position, the motor can be driven in the correct direction. All in all, the uncertainty can be removed by using the proposed idea.
6 Rotor Position Calculation

The flux linkage signal equation has been derived in the previous chapter, which are periodical signals and the phase shift between the signals is constant. These characteristics can be applied to calculate the electrical rotor position ($\alpha_{cal}$).

In this chapter, different rotor position calculation methods are described, derived and analyzed based on two criteria, i.e. relation of flux linkage signals and relation of phase inductances.

Finally, the rotor position calculation methods are implemented in the real time system to drive the motor. In order to figure out the accuracy of the estimation methods and also confirm that the calculated position is the exact rotor position, the estimated results are compared with the measured positions, which are attained by using an encoder transducer or a mechanical sensor.

6.1 Rotor Position Calculation Method

The rotor position calculation method has been firstly discussed in Chapter 3. The sinusoidal waveforms have been used as the assumed flux linkage signals. Presently, the finalized flux linkage signal equation has been stated in (5.55). The flux linkage signals can be calculated and depicted in Fig. 5.7 by assuming $L_d$, $L_q$ and $V_{DC}$ as 0.003 mH, 0.004 mH and 30 V, respectively.

In this part, the half of an electrical period, which is equal to one period of the flux linkage signal, is only used to investigate and estimate the rotor position. The one period of the flux linkage signals or the DFC signals by using the same parameters as in Chapter 5 are illustrated in Fig. 6.1. The signals are generated by increasing the electrical angle from 0 to $\pi$ rad linearly. Hence, the calculated rotor position must have the same linear characteristic.

There are two main approaches to calculate the electrical rotor postion, which are done by using the relation of flux linkage signals and the realtion of phase inductances. Each approach is described and discussed in this chapter. In the end of this part, all methods are summarized by considering the estimation errors.
Fig. 6.1: Calculated flux linkage signals (u: red, v: blue, w: green)

6.1.1 Relation of Flux Linkage Signals

The rotor position estimation methods by using the relation of flux linkage signals can be done by three methods i.e. calculation based on the highest value, calculation based on the lowest value, and using the trigonometric relation. The method calculation and the calculated position of each estimation method are explained and shown, respectively.

6.1.1.1 Calculation Based on the Highest Value

The flux linkage signals are the same signals, except the phase shift of 60 degrees. In this case, the rotor position calculation can be achieved in (6.1). The highest value is selected to be the denominator. The calculated results ($\alpha_{\text{cal,ph}}$) of each case are the repetitive values as described in Chapter 3.

$$
\alpha_{\text{cal,ph}} = \begin{cases} 
\frac{v-w}{u} : (u \geq v) \cap (u \geq w) \\
\frac{w-u}{v} : (v \geq u) \cap (v \geq w) \\
\frac{u-v}{w} : (w \geq u) \cap (w \geq v)
\end{cases}
$$

(6.1)
Each case has to be manipulated in the same way as applied in (3.20) to get the continual rotor position signal, which is represented in Fig. 6.2.

6.1.1.2 Calculation Based on the Lowest Value

This estimate method has been basically used for the DFC method. The calculation condition is only different from the previous calculation method. The lowest value is selected to be the denominator, which is stated in (3.20) and rewritten in (6.2).

\[
\alpha_{\text{cal,Pi}} = \begin{cases} 
\frac{v - w}{u} : (u \leq v) \cap (u \leq w) \\
\frac{w - u}{v} : (v \leq u) \cap (v \leq w) \\
\frac{u - v}{w} : (w \leq u) \cap (w \leq v)
\end{cases}
\]  

(6.2)

All repetitive results of \( \alpha_{\text{cal,Pi}} \) are also manipulated. After manipulating the position signals, \( \alpha_{\text{cal,Pi}} \) is depicted in Fig. 6.3. The difference between Fig. 6.2 and 6.3 can be fundamentally recognized by visualization. \( \alpha_{\text{cal,Pi}} \) is smoother than \( \alpha_{\text{cal,Ph}} \).
6.1.1.3 Using the Trigonometric Relation

The previous two estimation methods have been achieved by using the physical behaviors of the flux linkage signals. However, the trigonometric estimation method is computed by considering the finalized flux linkage signal equation in (5.55), rewritten in (6.3).

\[
\begin{align*}
    u &= -\xi \cos(2\alpha) + 0.5 \cos(4\alpha) \frac{V_{dc}}{3\xi^2 - 0.75} \\
    v &= \xi \cos(2\alpha + \frac{\pi}{3}) + 0.5 \cos(4\alpha + \frac{2\pi}{3}) \frac{V_{dc}}{3\xi^2 - 0.75} \\
    w &= \xi \cos(2\alpha - \frac{\pi}{3}) + 0.5 \cos(4\alpha - \frac{2\pi}{3}) \frac{V_{dc}}{3\xi^2 - 0.75}
\end{align*}
\] (6.3)

The summation of \( u, v, \) and \( w \) is zero. Then, there is an assumption in this calculation in (6.4). The absolute value of \( \xi \) is much more than 0.5. Then, the fourth harmonic parts are neglected, which leads to have the flux linkage signals equation in (6.5).

\[
|\xi| \gg 0.5
\] (6.4)
Rotor Position Calculation

\[
\begin{align*}
    u &\approx -\xi \cos(2\alpha) V_{dc} \\
    v &\approx \frac{\xi \cos(2\alpha + \frac{\pi}{3})}{3\xi^2 - 0.75} V_{dc} \\
    w &\approx \frac{\xi \cos(2\alpha - \frac{\pi}{3})}{3\xi^2 - 0.75} V_{dc}
\end{align*}
\]

(6.5)

Consequently, the flux linkage signals can be used to estimate the electrical rotor position by using trigonometric relations with two possibilities in (6.6) and (6.7).

\[
\alpha_{cal,PF} = \frac{1}{2} \angle(u \cdot e^{-j\xi} + v \cdot e^{-j\frac{\pi}{3}} + w \cdot e^{j\frac{\pi}{3}})
\]

(6.6)

\[
\begin{align*}
    \alpha_{cal,PF} &= \frac{1}{2} \arctan \left( \frac{v - w}{\sqrt{3}u} \right) \\
    &= \frac{1}{2} \arctan \left( \frac{\xi V_{dc} \left( \cos(2\alpha + \frac{\pi}{3}) - \cos(2\alpha - \frac{\pi}{3}) \right)}{-\sqrt{3}\xi V_{dc} \cos(2\alpha)} \right) \\
    &= \frac{1}{2} \arctan \left( \frac{-\sqrt{3} \sin(2\alpha)}{-\sqrt{3} \cos(2\alpha)} \right) \\
    &= \alpha
\end{align*}
\]

(6.7)

Regarding the assumed machine parameters, \( L_d \) and \( L_q \) are 0.003 mH, 0.004 mH. \( \xi \) can be calculated in (6.8). The calculated electrical rotor positions \( \alpha_{cal,PF} \) are displayed in Fig. 6.4.

\[
\xi = \frac{L_d}{L_q} = \frac{L_d + L_q}{L_d - L_q} = \frac{(3+4) \cdot 10^{-6} \text{H}}{(3-4) \cdot 10^{-6} \text{H}} = -7
\]

(6.8)

Actually, the trigonometric relation can be correctly estimated when the relation between \( \sqrt{3} u \approx -\sqrt{3} \cos(2\alpha) \) and \( (v - w) \approx -\sqrt{3} \sin(2\alpha) \) is a circle, which is the two dimensional relation (2D relation). The 2D relation of the used \( L_d \) and \( L_q \) is also shown in Fig. 6.5.
In order to assure the influence of $\xi$, $L_q$ is changed to 0.0033 mH. Thus, $\xi$ becomes $-21$. $\alpha_{cal,PF}$ and 2D relation of the modified $\xi$ are shown in Fig. 6.6 and 6.7.

$$\xi = \frac{L_x}{L_y} = \frac{L_d + L_q}{L_d - L_q} = \frac{(3 + 3.3) \cdot 10^{-6} \text{H}}{(3 - 3.3) \cdot 10^{-6} \text{H}} = -21$$

(6.9)
The results show that the estimated positions are much more linear and the 2D relation is almost in a circle shape, when the absolute of $\xi$ is much greater than 0.5 as assumed in (6.4).
6.1.2 Relation of Phase Inductances

The three rotor position estimation methods by using the relation of flux linkage signals have been explained. The differences between the phase inductances and the flux linkage signals can be simply found by the mathematic expression in (6.10) and (6.11). Concerning the second harmonic, the phase difference between \( u \) and \( L_U \), \( v \) and \( L_V \), and \( w \) and \( L_W \) is \( \frac{\pi}{2} \) rad, which is from the relation of the cosine function.

\[
L_U = \frac{1}{3} (L_x + L_y \cos(2\alpha))
\]
\[
L_V = \frac{1}{3} (L_x + L_y \cos(2\alpha + \frac{4\pi}{3})) \tag{6.10}
\]
\[
L_W = \frac{1}{3} (L_x + L_y \cos(2\alpha + \frac{2\pi}{3}))
\]

\[
u = \frac{-\xi \cos(2\alpha) + 0.5 \cos(4\alpha)}{3\xi^2 - 0.75} V_{dc}
\]
\[
v = \frac{\xi \cos(2\alpha + \frac{\pi}{3}) + 0.5 \cos(4\alpha + \frac{2\pi}{3})}{3\xi^2 - 0.75} V_{dc} \tag{6.11}
\]
\[w = \frac{\xi \cos(2\alpha - \frac{\pi}{3}) + 0.5 \cos(4\alpha - \frac{2\pi}{3})}{3\xi^2 - 0.75} V_{dc} \]

The finalized flux linkage signals (6.11) are basically derived from the normalized flux linkage signal (5.46), rewritten in (6.12).

\[
u = \frac{V_{dc}}{L_U} + \frac{1}{L_V} + \frac{1}{L_W} - \frac{1}{3} V_{dc}
\]
\[
u = \frac{V_{dc}}{L_U} + \frac{1}{L_V} + \frac{1}{L_W} - \frac{1}{3} V_{dc} \tag{6.12}
\]
\[w = \frac{L_V}{L_U} + \frac{1}{L_V} + \frac{1}{L_W} - \frac{1}{3} V_{dc}
\]
\[w = \frac{L_W}{L_U} + \frac{1}{L_V} + \frac{1}{L_W} - \frac{1}{3} V_{dc} \]
If the offset term ($-\frac{1}{3}V_{DC}$) is removed, the none offset flux linkage signal can be stated in (6.13). The position signal ($PL_{U, raw}$, $PL_{V, raw}$, $PL_{W, raw}$) of each phase can be simply found in (6.14).

\[
U = \frac{V_{DC}}{L_u + \frac{1}{L_v} + \frac{1}{L_w}} = \frac{L_u L_w}{L_u L_v + L_v L_w + L_w L_u} V_{DC}
\]

\[
V = \frac{V_{DC}}{L_v} = \frac{L_w L_v}{L_u L_v + L_v L_w + L_w L_u} V_{DC}
\]

\[
W = \frac{V_{DC}}{L_w} = \frac{L_u L_v}{L_u L_v + L_v L_w + L_w L_u} V_{DC}
\]

\[
PL_{U, raw} = \sqrt{\frac{L_u L_v L_w^2}{u}} \frac{V_{DC}}{L_u L_v + L_v L_w + L_w L_u} = L_u \sqrt{\frac{V_{DC}}{(L_u L_v + L_v L_w + L_w L_u)}}
\]

\[
PL_{V, raw} = \sqrt{\frac{L_w L_v L_u^2}{v}} \frac{V_{DC}}{L_u L_v + L_v L_w + L_w L_u} = L_v \sqrt{\frac{V_{DC}}{(L_u L_v + L_v L_w + L_w L_u)}}
\]

\[
PL_{W, raw} = \sqrt{\frac{L_u L_v L_w^2}{w}} \frac{V_{DC}}{L_u L_v + L_v L_w + L_w L_u} = L_w \sqrt{\frac{V_{DC}}{(L_u L_v + L_v L_w + L_w L_u)}}
\]

It is worth to mention that all values in (6.14) have to be positive values; consequently the complex values cannot be produced. The multiplication factor ($k$) in (6.15) shows that $\xi^2$ must be greater than 0.25 in order to fulfill the condition.

\[
k = \sqrt{\frac{V_{DC}}{(L_u L_v + L_v L_w + L_w L_u)}} = \sqrt{\frac{V_{DC}}{3\xi^2 - 0.75}}
\]

Besides, the multiplication factor is also constant. Thus, $PL_{U, raw}$, $PL_{V, raw}$ and $PL_{W, raw}$ are calculated in (6.16), which are also depicted in Fig. 6.8. The unit of the position signal is $V^{1/2}$. 
Rotor Position Calculation

\[
P_{L_{U,\text{raw}}} = kL_U = \frac{k}{3} (L_x + L_y \cos(2\alpha))
\]

\[
P_{L_{V,\text{raw}}} = kL_V = \frac{k}{3} (L_x + L_y \cos(2\alpha + \frac{4\pi}{3}))
\]

\[
P_{L_{W,\text{raw}}} = kL_W = \frac{k}{3} (L_x + L_y \cos(2\alpha + \frac{2\pi}{3}))
\]

(6.16)

Fig. 6.8: Position signals (\(PL_{U,\text{raw}}\) : red, \(PL_{V,\text{raw}}\) : blue, \(PL_{W,\text{raw}}\) : green)

In order to calculate the electrical rotor position, there are two possibilities. Firstly, the estimated rotor positions by the phase inductances (\(\alpha_{\text{cal,PL}}\)) can be found by using the Euler relation in (6.17).

\[
\alpha_{\text{cal,PL}} = \frac{1}{2} \angle (P_{L_{U,\text{raw}}} + P_{L_{V,\text{raw}}} e^{-j\frac{2\pi}{3}} + P_{L_{W,\text{raw}}} e^{-j\frac{2\pi}{3}})
\]

(6.17)

Secondly, the offsets in the position phase inductance signals have to be removed and the trigonometric relation can be applied to estimate the electrical rotor position. The offsets can be eliminated by using the constant values in (6.18) and calculate in (6.19) as shown in Fig. 6.9.

\[
P_{L_{\text{sum}}} = \frac{P_{L_{U,\text{raw}}} + P_{L_{V,\text{raw}}} + P_{L_{W,\text{raw}}}}{3} = \frac{kL_x}{3}
\]

(6.18)
Rotor Position Calculation

\[
\begin{align*}
PL_U &= PL_{U,\text{raw}} - PL_{\text{sum}} = \frac{kL_y}{3} \cos(2\alpha) \\
PL_V &= PL_{V,\text{raw}} - PL_{\text{sum}} = \frac{kL_y}{3} \cos(2\alpha + \frac{4\pi}{3}) \\
PL_W &= PL_{W,\text{raw}} - PL_{\text{sum}} = \frac{kL_y}{3} \cos(2\alpha + \frac{2\pi}{3})
\end{align*}
\]

\[(6.19)\]

Fig. 6.9: Position signal without offsets \((PL_U: \text{red}, PL_V: \text{blue}, PL_W: \text{green})\)

Accordingly, \(\alpha_{\text{cal,PL}}\) can be estimated in (6.20). \(\alpha_{\text{cal,PL}}\) and the 2D relation of \(\alpha_{\text{cal,PL}}\) can be found and depicted in Fig. 6.10 and 6.11, respectively. In Fig. 6.10, \(\alpha_{\text{cal,PL}}\) is added by \(\frac{\pi}{2}\) rad in order to have the same range of the results as \(\alpha_{\text{cal,PF}}\) from 0 to \(\pi\) rad.

\[
\alpha_{\text{cal,PL}} = \frac{1}{2} \arctan\left(\frac{PL_V - PL_W}{\sqrt{3}PL_U}\right)
\]

\[
= \frac{1}{2} \arctan\left(\frac{\frac{kL_y}{3} \cos(2\alpha + \frac{4\pi}{3}) - \cos(2\alpha + \frac{2\pi}{3})}{\sqrt{3} \frac{kL_y}{3} \cos(2\alpha)}\right)
\]

\[
= \frac{1}{2} \arctan\left(\frac{\sqrt{3} \sin(2\alpha)}{\sqrt{3} \cos(2\alpha)}\right)
\]

\[
= \alpha
\]

\[(6.20)\]
Fig. 6.10: $\alpha_{\text{cal,PL}}$

Fig. 6.11: 2D relation of position signals

Fig. 6.11 shows that the 2D relation by using the position signals based on the relation of the phase inductances is much more in the circle shape than using the relation of the flux linkage signals. This is because the fourth harmonic is completely eliminated by the calculation in (6.14).
Furthermore, one of the most important steps is to remove the offset terms $-\frac{1}{3}V_{dc}$ in (6.12) to be (6.13). They can be easily removed by adding $\frac{1}{3}V_{dc}$. If another value is used instead, the fourth harmonic will occur. The investigation to confirm the mentioned statement has been done. The offset value has been varied in order to add to (6.13) and calculate $PL_{U,\text{raw}}$, $PL_{V,\text{raw}}$ and $PL_{W,\text{raw}}$ in (6.14). The spectrum ratio between the fourth and the second harmonics of the position signals in (6.14) have been captured, while varying the offset. The relation between the offset and the spectrum ratio can be represented in Fig. 6.12. The smallest spectrum ratio is located at 10, which is equal to $\frac{1}{3}V_{dc}$ ($V_{DC} = 30$ V).

Therefore, the proper offset ($V_{\text{offset}}$) is required in order to implement the rotor position calculation by using the relation of the phase inductances. Otherwise, the 2D relation circle shape can be distorted, which means the increasing of the fourth harmonic, similar to the distorted 2D relation in Fig. 6.5 and 6.7.

![Fig. 6.12: Influence of offset through spectrum ratio of position signals](image-url)
6.1.3 Summary

The electrical rotor position ($\alpha_{\text{cal}}$) has been estimated by four methods based on two criteria. The exact electrical rotor position ($\alpha$) is set to increase from 0 to $\pi$, linearly. Therefore, the accuracy of the calculation methods can be found by using the estimation error, which can be computed in (6.21).

$$\text{err}_{\alpha_{\text{cal}}} = \alpha - \alpha_{\text{cal}}$$ (6.21)

In this case, all calculated positions have been manipulated to be in the same range from 0 to $\pi$ rad. Thus, all estimation errors of four methods can be calculated and depicted in Fig. 6.13. The error characteristics can be concluded in (6.22).

$$\text{err}_{\alpha_{\text{cal,Ph}}} > \text{err}_{\alpha_{\text{cal,Pf}}} > \text{err}_{\alpha_{\text{cal,PF}}} > \text{err}_{\alpha_{\text{cal,PL}}}$$ (6.22)

![Fig. 6.13: Error of estimated rotor positon](image)

It is noteworthy that $\text{err}_{\alpha_{\text{cal,Pf}}}$ can be decreased, when the absolute value of $\xi$ is bigger, which leads to have the 2D relation close to the circle shape.
As results, two of the rotor position calculation methods, which make small errors, \( \alpha_{\text{cal PF}} \) and \( \alpha_{\text{cal PL}} \), are selected to implement in the real time system to assure the calculated positions and the rotor calculation method capability.

### 6.2 Real Time Implementation of Position Calculation

In this part, two rotor position estimation methods, \( \alpha_{\text{cal PF}} \) and \( \alpha_{\text{cal PL}} \), are applied to drive the motor in the real time system. The hardware environment has been prepared in order to use the calculated rotor position as the exact position and comparing with the mechanical rotor position (\( \alpha_R \)), which is obtained by a mechanical sensor.

Moreover, all discussed aspects e.g. finding the proper \( V_{\text{offset}} \), 2D relation, flux linkage signals spectrum of two position calculation techniques are also employed and analyzed in the experimental setup.

#### 6.2.1 Hardware Environment

First of all, the tested motor is a new motor, PMSM4, which is depicted in Fig. 6.14. PMSM4 has an out stator with 12 stator teeth and 22 rotor magnets or 11 permanent magnet pole pairs. The maximum \( V_{\text{DC}} \) for PMSM4 is 50V and the electrical power is around 1 kW. The phase inductance varies between 64.2 \( \mu \text{H} \) and 72.1 \( \mu \text{H} \). The rotor flux (\( \Psi_r \)) is 6.24 mVs.

![Fig. 6.14: PMSM4](image)
PMSM4 has been installed to a new test bench for big machines as shown in Fig. 6.15. Several parts of electrical measurements circuits have been developed in [51] in order to prevent electromagnetic interferences (EMI).

**Fig. 6.15: New test bench for big machines**

PMSM4 is connected to the TriCore PXROS platform with a PWM frequency of 10 kHz and $V_{DC}$ is set to 30 V. The measuring time after switching on the pulse ($t_m$) is 2700 ns, the same as for PMSM2. $V_{NAN}$ is measured by the modified measuring sequence.

The load motor is the Stromag motor, which is driven by a DriveStar inverter system. The DriveStar system also provides the mechanical rotor position by converting the sin/cos encoder signals to incremental encoder signals. The incremental encoder signals are used as the input of the TriCore PXROS platform. There are two incremental encoder signals, i.e. A and B, which are pulse train signals. The resolution is 512 pulses per one mechanical round. The pulse trains of A and B are depicted in Fig. 6.16. $POS$ is the counter signal, which is counted whenever any signal is changed. The direction can be detected by checking the change sequence, e.g. $ABABAB$ is the positive direction. Thus, one mechanical round has 2048 changes (0 to 2047, 11 bits data). The mechanical rotor position ($\alpha_r$) can be found in (6.23).

$$\alpha_r = \frac{POS}{2047} \cdot 2\pi \quad (6.23)$$
6.2.2 Experimental Setup

Regarding the DFC signals or the flux linkage signals, they can be obtained when $L_d$ and $L_q$ are more or less invariable. The flux linkage signals can be distorted, if the voltage input vector generates either the field weakening or the field strengthening through the rotor flux. Therefore, the input voltage vector must be always aligned on the $q$ axis. Subsequently, in order to fulfill the mentioned characteristic the DFC input voltage space vector by using $\alpha_{cal}$ as the electrical rotor position has been derived in appendix 10.3 and can be concluded in (6.24), where the electrical correction angle ($\alpha_k$) or the commutation advance angle is set to zero.

\[
\begin{bmatrix}
V_d \\ V_q
\end{bmatrix} = \begin{bmatrix}
\frac{3}{2} \frac{P_m}{100} V_{dc} \sin(\alpha_k) \\ -\frac{3}{2} \frac{P_m}{100} V_{dc} \cos(\alpha_k)
\end{bmatrix}
\]

(6.24)
Fundamentally, $\alpha_{\text{cal,PF}}$ can be directly found by using the extracted flux linkage signals from $V_{\text{NAN}}$. However, the flux linkage signals have to be firstly reconstructed to the position signal of the phase inductances ($PL_U$, $PL_V$, $PL_W$), with several required steps before calculating $\alpha_{\text{cal,PL}}$. Moreover, the relation between $\alpha_R$ and $\alpha_{\text{cal}}$ has to be found before assuring the calculated rotor position. Both steps are explained, correspondingly.

### 6.2.2.1 Setup $\alpha_{\text{cal,PL}}$ Calculation

The flux linkage signals, i.e. $u$, $v$, and $w$, which have been already normalized, can be obtained from the system. In order to reconstruct the flux linkage signals that can be used to calculate the position signals of the phase inductances, $V_{\text{offset}}$ has to be found. Although the proper $V_{\text{offset}}$ has been defined by $\frac{1}{3}V_{\text{DC}}$, it is difficult to apply in the real time system. This is because $V_{\text{NAN}}$ has been processed e.g. damped and amplified in several steps. Thus, the proper $V_{\text{offset}}$ can be found by varying $V_{\text{offset}}$ and figure out the lowest value of the spectrum ratio between the fourth and the second harmonics of the position signals.

In order to get the flux linkage signals without any influence of the driving system, $P_M$ is set to 3%, which the switch on time duration is equal to 3000 ns with the PWM frequency of 10 kHz, for all three phases and $V_{\text{NAN}}$ is measured by the modified measuring sequence. The driving pulses are fixed and independent from the calculated rotor position.

![Diagram](image)

**Fig. 6.17: Coupled shaft motor**

The PMSM4 shaft is coupled with the load motor as in Fig. 6.17. The load motor is driven at a constant speed in this case at 60 rpm. Then, the flux linkage signals have
been collected. For instance, one electrical period of the PMSM4 flux linkage signals is depicted in Fig. 6.18.

Fig. 6.18: PMSM4 flux linkage signals at 60 rpm (u: red, v: blue, w: green)

Fig. 6.18 shows that the flux linkage signals are varied in the range of -40 mV to 40 mV, approximately. Thus, the varied $V_{offset}$ begins from 40 mV in order to have only the positive values in the calculation as in (6.25). Then, the spectrum ratio between the fourth and the second harmonic of the position signal in (6.26) are captured, while varying $V_{offset}$.

\[
\begin{align*}
  u &= u + V_{offset} \\
  v &= v + V_{offset} \\
  w &= w + V_{offset}
\end{align*}
\]  \tag{6.25}

\[
\begin{align*}
  PL_{u, raw} &= \sqrt{\frac{vw}{u}} \\
  PL_{v, raw} &= \sqrt{\frac{uw}{v}} \\
  PL_{w, raw} &= \sqrt{\frac{uv}{w}}
\end{align*}
\]  \tag{6.26}
The influence of $V_{\text{offset}}$ through the PMSM4 flux linkage signal spectrum has been found in Fig. 6.19. Consequently, the proper $V_{\text{offset}}$ of PMSM4 for the new test bench is 135 mV, where the spectrum ratio is 0.013.

After applying the found $V_{\text{offset}}$, the position signal of the phase inductances can be calculated. It is noteworthy that the variable type in the calculation is integer; the decimal numbers are automatically neglected. Hence, a gain of 1000 is applied for the position signal calculation in order to increase the number resolution. The gain can be applied in (6.27).

\[
\begin{align*}
PL_{u, \text{raw}} &= \sqrt{1000 \cdot \frac{vw}{u}} \\
PL_{v, \text{raw}} &= \sqrt{1000 \cdot \frac{uw}{v}} \\
PL_{w, \text{raw}} &= \sqrt{1000 \cdot \frac{uv}{w}} .
\end{align*}
\]  

(6.27)

After removing the constant offset of the position signals by using $PL_{\text{sum}}$ in (6.18), the position signals and the flux linkage signals of each phase in one electrical period are depicted in Fig. 6.20 to 6.22, respectively.
Fig. 6.20: $u$ and $PL_U$ at 60 rpm

Fig. 6.21: $v$ and $PL_V$ at 60 rpm
According to Fig. 6.20 to 6.22, it can be found that the flux linkage signals and the phase inductance signals have the phase difference at \( \pi \) rad as in (6.28). The phase difference is \( \frac{\pi}{2} \) rad, if the fundamental frequency is taken into account. This is because both flux linkage signal and position signal are second harmonic signals.

\[
\pi = \frac{\angle u - \angle PL_u}{\angle v - \angle PL_v} = \frac{\angle w - \angle PL_w}{\angle v - \angle PL_v} \tag{6.28}
\]

Besides, the flux linkage signal \((u)\) and the position signal \((PL_U)\) are also analyzed by calculating the spectrums, which are displayed in Fig. 6.23 and 6.24. PMSM4 has 11 permanent magnet pole pairs. It is driven by the loaded PMSM at 60 rpm, which means the fundamental electrical frequency is 11 Hz. The second and the fourth harmonics are 22 and 44 Hz, respectively. As results, the difference between the spectrums of the two signals is that the spectrum of \(PL_U\) does not have the fourth harmonic.

Next, the 2D relation of the flux linkage signals and the position signals are considered. Each 2D relation is represented in Fig. 6.25 and 6.26, respectively.
Fig. 6.23: Spectrum of at 60 rpm

Fig. 6.24: Spectrum of $PL_U$ at 60 rpm
Fig. 6.25: 2D relation of flux linkage signals at 60 rpm

Fig. 6.26: 2D relation of position signals at 60 rpm
After investigating all required properties for the rotor position calculated by using the relation of the phase inductances $\alpha_{\text{cal,PL}}$, PMSM4 is driven by the load motor at the same constant speed, the same measuring pulses are also applied. Both rotor position calculation techniques are implemented. $\alpha_{\text{cal,PF}}$ and $\alpha_{\text{cal,PL}}$ are depicted in Fig. 6.27. The difference between two calculated positions is also shown, which is constant at $\frac{\pi}{2}$ rad, approximately. The relation between $\alpha_{\text{cal,PF}}$ and $\alpha_{\text{cal,PL}}$ is stated in (6.29).

$$\alpha_{\text{cal,PL}} \approx \alpha_{\text{cal,PF}} - \frac{\pi}{2}$$  \hspace{1cm} (6.29)

Consequently, $\alpha_{\text{cal,PL}}$ must be added by $\frac{\pi}{2}$ rad in order to have the voltage vector aligned on the $q$ axis, when this calculation technique is selected to implement on the system. The reason can be explained in (6.30), which is briefly derived based on the derivation in appendix 10.3. $\alpha_k$, which is another input of the DFC method as shown...
Rotor Position Calculation

in Fig. 3.26 and generally used for adjusting the magnetic field by stator currents, can be also adapted to use in order to add $\frac{\pi}{2}$ to the voltage vector calculation.

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \frac{3}{2} P_m V_{DC} \sin((\alpha_{cal,PF} - \frac{\pi}{2}) + \frac{\pi}{2} - \alpha_{cal}) \\ -\frac{3}{2} P_m V_{DC} \cos((\alpha_{cal,PF} - \frac{\pi}{2}) + \frac{\pi}{2} - \alpha_{cal}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} P_m V_{DC} \sin(0) \\ -\frac{3}{2} P_m V_{DC} \cos(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\frac{3}{2} P_m V_{DC} \end{bmatrix}$$

(6.30)

After adding $\frac{\pi}{2}$ rad to $\alpha_{cal,PL}$, the differences between $\alpha_{cal,PL}$ and $\alpha_{cal,PF}$ have been found and represented in Fig. 6.28. The differences are in between -3 to 3 degrees. The reason to calculate the difference is to recheck that the difference trend is the same as the $err\alpha_{cal,PF}$ characteristics in Fig. 6.13, if $\alpha_{cal,PL}$ is the correct rotor position.

Fig. 6.28: $\alpha_{cal,PL} - \alpha_{cal,PF}$
All in all, the setup for the $\alpha_{cal,PL}$ calculation has been done. The proper $V_{offset}$ has been found. The phase difference between the flux linkage signal and the position signal is constant at $\pi$ rad. The fourth harmonic in the position signals is eliminated by the proposed calculation. The 2D relation of the position signals is much more in the circle shape than the 2D relation of the flux linkage signals. The difference between $\alpha_{cal,PL}$ and $\alpha_{cal,PF}$ in the real time system is also the same as in the error analysis. The requirement to add $\frac{\pi}{2}$ rad in order to use $\alpha_{cal,PL}$ to drive the motor with the DFC method is also elucidated. It can be concluded that all implementation aspects conform to the theoretical aspects.

### 6.2.2.2 Relation between Mechanical Position and Calculated Electrical Position

Principally, the relation between the mechanical rotor position ($\alpha_R$) and the electrical rotor position ($\alpha$) is in (6.31), where $p_R$ is number of permanent magnet pole pairs.

$$\alpha_R = p_R \alpha$$

(6.31)

The mechanical rotor position can be attained by using the incremental encoder signals. The resolution of the counter is 2048 counts or 11 bits per one mechanical round.

Nevertheless, the calculated electrical rotor position ($\alpha_{cal}$) signal has 1024 counts or 10 bits resolution per one electrical period, which is fixed by an array size of the trigonometric functions in software.

Consequently, the proper way to compare between $\alpha_R$ and $\alpha_{cal}$ can be done by converting $\alpha_{cal}$ to be the estimated mechanical position $\alpha_{R,cal}$. This is because the higher resolution signal is appropriate to adjust the resolution to be the same as the lower resolution signal in order to avoid losing any information by modifying the less resolution signal as $\alpha_R$.

For instance, $p_R$ of PMSM4 is 11 and PMSM4 is driven by the load motor at 60 rpm, 1 round per second (1Hz). $\alpha_R$ and $\alpha_{cal}$ have been captured and displayed in Fig. 6.29.
Rotor Position Calculation

Eleven periods of $\alpha_{cal}$ is one period of $\alpha_R$, which can confirm the definition in (6.31). By using numerical techniques, i.e. normalization and rounding function, $\alpha_{cal}$ can be transformed to $\alpha_{R,cal}$, which has the same resolution as $\alpha_R$.

Fig. 6.29: Mechanical rotor position estimation $\alpha_{R,cal}$

6.2.3 Experimental Results and Analysis

Two experiments have been done. The first experiment uses $\alpha_{cal,PF}$ to drive the system. Another experiment is achieved by using $\alpha_{cal,PL}$ as the electrical rotor position to run the system. Both experiments have been performed by applying $P_M$ at 25%. The PMSM4 mechanical speed is in the region of 280 rpm. $\alpha_R$ and $\alpha_{cal}$ of each experiment are taken into account, which are represented in Fig. 6.30 by using $\alpha_{cal,PF}$ and in Fig. 6.31 by using $\alpha_{cal,PL}$.

The calculated electrical rotor positions are converted to the estimated mechanical rotor positions in Fig. 6.32. The errors of the estimated mechanical rotor position in rad can be found in (6.32) and are depicted in Fig. 6.33.
\[
\text{err} \alpha_{R, \text{cal, PF}} = \frac{(\alpha_R - \alpha_{R, \text{cal, PF}})}{2047} \cdot 2\pi \\
\text{err} \alpha_{R, \text{cal, PL}} = \frac{(\alpha_R - \alpha_{R, \text{cal, PL}})}{2047} \cdot 2\pi
\]  

(6.32)

Fig. 6.30: $\alpha_R$ and $\alpha_{\text{cal, PF}}$ ($P_M = 25\%$)

Fig. 6.31: $\alpha_R$ and $\alpha_{\text{cal, PL}}$ ($P_M = 25\%$)
Fig. 6.32: $\alpha_{R,\text{cal},PF}$ and $\alpha_{R,\text{cal},PL}$ ($P_M = 25\%$)

Fig. 6.33: $err\alpha_{R,\text{cal},PF}$ and $err\alpha_{R,\text{cal},PL}$ [Degree] ($P_M = 25\%$)
In order to define the calculation method capability, the root mean square (rms) errors in Fig. 6.33 have been calculated.  

\[ \text{err}_{R,\text{cal},PF}(\text{rms}) \]  

and  

\[ \text{err}_{R,\text{cal},PL}(\text{rms}) \]  

are 0.7799 and 0.5384 degrees, respectively.

Indeed, both errors are very small values, which means that the calculated electrical rotor position by both methods, i.e. \( \alpha_{\text{cal},PF} \) and \( \alpha_{\text{cal},PL} \), can be assured that they are the exact rotor position and can be directly used in the further applications, e.g. field orient control (FOC).

It is worth to mention that the initial position of \( \alpha_R \) is always set to zero, which is different from \( \alpha_{\text{cal}} \). The initial position of \( \alpha_{\text{cal}} \) is found by the rotor position calculation method. Consequently, the experimental results in Fig. 6.30 and 6.31 show that the initial positions of \( \alpha_{\text{cal},PF} \) and \( \alpha_{\text{cal},PL} \) are different.

As a result, \( \alpha_{\text{cal},PF} \) and \( \alpha_{\text{cal},PL} \) can be used as the exact electrical rotor position. There are two more aspects based on the found spectrums in Fig. 6.23 and 6.24, that must be analyzed. At first, the spectrum of the flux linkage signal shows that the second harmonic is much greater than the fourth harmonic, and the spectrum ratio between the fourth and the second harmonics is 0.0886. It means that the absolute inductance ratio (\( \xi \)) of PMSM4 is much bigger than 0.5.

To provide the evidence of the \( \xi \) value, the phase inductance of PMSM4 has been stated, which varies between 64.2 µH \( (L_{p(min)}) \) and 72.1 µH \( (L_{p(max)}) \). The phase inductance approximation function of phase \( U \) is in (6.33). Consequently, \( L_d \) and \( L_q \) can be found when \( \alpha \) is at 0 and \( \frac{\pi}{2} \) rad as in (6.34). This is because in the case of PMSM, \( L_d \) is always less than \( L_q \) ([37], [48]).

\[
L_U = \frac{1}{3} \left( (L_d + L_q) + (L_q - L_d) \cos(2\alpha) \right) \tag{6.33}
\]

\[
L_{U,(\text{min})} = \frac{2}{3} L_d \quad : \quad \alpha = 0
\]

\[
L_{U,(\text{max})} = \frac{2}{3} L_q \quad : \quad \alpha = \frac{\pi}{2} \tag{6.34}
\]
Therefore, the phase inductance is illustrated in Fig. 6.34. $L_d$ and $L_q$ are 96 $\mu$H and 108.15$\mu$H, respectively. The inductance ratio ($\xi$) of PMSM4 is calculated in (6.35), which matches to the definition pointed out.

$$\xi = \frac{L_x}{L_y} = \frac{L_d + L_q}{L_d - L_q} = \frac{(96 + 108.15) \cdot 10^{-6} \text{H}}{(96 - 108.15) \cdot 10^{-6} \text{H}} = -16.8$$

Secondly, the spectrum at the fundamental frequency (11 Hz) occurs in both signals spectrums. Even though the spectrum at 11 Hz is quite small and it does not influence to the DFC method, the first harmonic should not turn out.

After considering the system and the DFC conditions, it has been found that PMSM4 is an unbalanced motor. The experiments to assure the unbalance characteristic by the motor itself have been done and explained in appendix 10.4. Due to that, the motor phase currents become asymmetric, which leads to have the inconstant current vector in the synchronous frame. The inconstant current vector directly influences the fluxes and the inductances in the synchronous frame. Subsequently, the fundamental frequency can be coupled into the flux linkage signals.
7 Sensorless Closed Loop Speed Control with DFC

The sensorless closed loop speed control is usually achieved by a field oriented control (FOC) approach and it is basically done in the \((d, q)\) frame. Therefore, the machine model in the \((d, q)\) frame has to be considered. The PMSM time domain differential equation is stated in (7.1) [8].

\[
\begin{align*}
\frac{dI_d}{dt} &= \frac{-R_d I_d + p_R \omega_R L_q I_q + V_d}{L_d} \\
\frac{dI_q}{dt} &= \frac{-p_R \omega_R L_q I_d - R_d I_q + p_R \omega_R \Psi_f + V_q}{L_q} \\
\frac{d\omega_R}{dt} &= \frac{p_R (L_d - L_q) I_q I_d + p_R \Psi_f I_q - T_L}{J} \\
\frac{d\alpha}{dt} &= p_R \omega_R
\end{align*}
\]

\((\omega_R)\) is the mechanical frequency [rad/s], \(T_L\) is the torque load (disturbance) in Nm and \(J\) is the moment of inertia in kgm\(^2\). Based on the model, the control parameters can be simply found. For instance, the time constant in \(q\) axis \((\tau_q)\) can be found by locking the rotor and applying a \(V_q\) step input.

The exact electrical rotor position can be obtained by the DFC method. As investigated, there are two estimation techniques, i.e. using the relation of flux linkage signals \((\alpha_{cal,PF})\) and using the relation of phase inductances \((\alpha_{cal,PL})\). Both methods have been confirmed and also provide the same results. Thus, the calculated rotor position for the closed loop speed control implementation is represented by \(\alpha_{cal}\) in order to refer to either \(\alpha_{cal,PF}\) or \(\alpha_{cal,PL}\).

Consequently, the sensorless closed loop speed control is needed to setup first. The system structure has to be analyzed in order to combine DFC and FOC together, and also the control parameters have to be found and adjusted. Then, the setup system is utilized to perform the experiments in order to demonstrate the sensorless closed loop speed control with DFC for all speeds. Each part is described, respectively.
7.1 Closed Loop Speed Control Setup

In order to design the closed loop speed control, there are three main loops, which are taken into account, i.e. internal, central and external loops. The loops are named, i.e. DFC structure, current control loop and speed control loop, correspondingly. The explanation and investigation of each loop is described from the internal loop to the external loop as following.

7.1.1 DFC Structure

The DFC structure is the internal loop. This is because the DFC method has united the PWM unit and the position calculation as depicted in Fig. 7.1. The DFC inputs are \( P_M \) and \( \alpha_k \).

![DFC Diagram](image)

*Fig. 7.1: DFC Diagram*

The FOC scheme requires the currents \( (I_d, I_q) \), which can be converted from the phase currents \( (I_U, I_V, I_W) \). Thus, the phase currents are measured as the outputs of the internal loop. Therefore, the calculation to obtain the proper inputs and outputs are given details.

7.1.1.1 DFC Inputs

As derived in appendix 10.3 and explained in (6.24), \( V_d \) and \( V_q \), and \( \alpha_k \) can be found in (7.2) and (7.3). The resultant voltage \( |V| \) in (7.4) can be used to find \( P_M \) as in (7.5).
Sensorless Closed Loop Speed Control with DFC

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} = \begin{bmatrix}
\frac{3}{\sqrt{2}} \frac{P_M}{100} V_{dc} \sin(\alpha_k) \\
-\frac{3}{\sqrt{2}} \frac{P_M}{100} V_{dc} \cos(\alpha_k)
\end{bmatrix}
\]  

(7.2)

\[
\alpha_k = -\arctan\left(\frac{V_q}{V_d}\right)
\]

(7.3)

\[
|V| = \sqrt{V_d^2 + V_q^2}
\]

\[
= \sqrt{\left(\frac{3}{2} \frac{P_M}{100} V_{dc}\right)^2 (\sin^2(\alpha_k) + \cos^2(\alpha_k))}
\]

(7.4)

\[
P_M = \frac{2}{3} \frac{|V|}{V_{dc}} \times 100
\]

(7.5)

### 7.1.1.2 DFC Outputs

The three phase currents are measured in order to convert to the currents in \((d, q)\) frame. Based on the derivation in appendix 10.3, \(I_q\) becomes minus, when the rotor is being moved in the positive direction (clockwise, viewing from in front of the motor).

Hence, the conversion matrix for the phase currents has to be modified. The investigation has been done in appendix 10.5. The new conversion matrix is in (7.6). The sign of \(I_q\) turns out to be the same sign of direction or the speed. Actually, there are also two possibilities to calculate the currents in stationary frame \((I_\alpha, I_\beta)\) in (7.7) and (7.8), which are clarified in appendix 10.6.

\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} = \begin{bmatrix}
-\sin(\alpha_{cal}) & -\cos(\alpha_{cal}) \\
-\cos(\alpha_{cal}) & \sin(\alpha_{cal})
\end{bmatrix} \begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix}
\]

(7.6)

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}}
\end{bmatrix} \begin{bmatrix}
I_U \\
I_V \\
I_W
\end{bmatrix}
\]

(7.7)
The calculation in (7.8) shows that only two phase currents can be utilized to convert to stationary frame.

The phase currents are measured as illustrated in Fig. 7.2. A structure of the phase current measurement is also depicted in Fig. 7.3. A current sensor in the system is the magneto-resistive current sensor (CDS4000 by Sensitec GmbH), which is designed for highly dynamic electronic measurement of DC, AC, pulses and mixed currents with integrated galvanic isolation. The current sensor output is a voltage signal ($V_{lp,raw}$), which must be processed in several steps, i.e. filtering, conditioning and amplifying, to be the appropriate voltage signal ($V_{lp}$) before feeding into the microcontroller.

**Fig. 7.2: DFC diagram with currents measurement**

**Fig. 7.3: Structure of current measurement**
The ADC unit of the microcontroller is used to convert the $V_{ip}$ signal to be a current signal ($I_{p,raw}$) in the system. Each phase is connected to each ADC port with 12 bits resolution and 3 µs conversion time. $V_{ip}$ is measured after all PWM signals are switched on or at $t_I$ of the modified measuring sequences as represented in Fig. 7.4. Since the voltage time areas of the voltage space vector in each period of the modified measuring sequences are the same, the sequence to attain the current signal of each ADC is unnecessary to be fixed. For instance, the measuring periods $T_1$, $T_2$ and $T_3$ in Fig. 7.4 are set to enable the ADCs in order to convert $V_{l_u}$, $V_{l_v}$ and $V_{l_w}$ to be $I_{U,raw}$, $I_{V,raw}$ and $I_{W,raw}$ respectively as in (7.9).

$$
I_{U,raw} = V_{l_u} \cdot t_I \\
I_{V,raw} = V_{l_v} \cdot t_I \\
I_{W,raw} = V_{l_w} \cdot t_I
$$

(7.9)

![Fig. 7.4: Phase currents measuring sequences](image)

After obtaining the $I_{p,raw}$ signal, a gain to get the real current value ($I_p$) has to be figured out by comparing the $I_{p,raw}$ values with a standard measurement while driving a machine. A multi-meter by FLUKE (89 IV) is used as the standard measurement and the found gain for the system is 6.08. $I_p$ can be calculated in (7.10).

$$
I_p = 6.08 I_{p,raw}
$$

(7.10)
7.1.1.3 Internal Loop Scheme

In this scheme setup, PMSM4 is selected to progress on the closed loop sensorless control. $V_{DC}$ is set at 40 V. The $V_{DC}$ value is directly related to calculate $\alpha_{cal,PL}$, the new $V_{offset}$ has to be found. The proper new $V_{offset}$ is 193 mV, which is from the same investigational strategy for $V_{offset}$ (135 mV) at $V_{DC}$ 30 V. The offset value for 40 V is 1.4 times of the old value, which is closed to 1.33 times of the $V_{DC}$ values. Thus, the definition of $V_{offset}$ can be directly applied to find the new $V_{offset}$s, when the $V_{DC}$ level in the same system is changed. However, $\alpha_{cal,PF}$ can be straight used without any modification.

Afterwards, PMSM4 is driven by applying $P_M$ at 50% and $\alpha_k$ is zero. The phase current is also measured, $I_{U(rms)}$ is 5.6 A, approximately. Then, $V_q$ and $I_q$ can be computed in (7.11) and (7.12). The relation between $V_q$ and $I_q$ is represented in (7.13), where $V_{q,s}$ is defined as $I_q$ with a gain of 1. It can be also used for the $d$ axis values.

\[
|V_q| = \sqrt{3} \frac{P_M}{2 \times 100} V_{DC} = 24.49 \text{ V} \quad (7.11)
\]

\[
|I_q| = \sqrt{3} I_{U(rms)} \cdot \sqrt{2} = 9.69 \text{ A} \quad (7.12)
\]

\[
V_q = 2.52V_{q,s} = 2.52 \frac{V}{A} \cdot I_q
\]

\[
V_d = 2.52V_{d,s} = 2.52 \frac{V}{A} \cdot I_d \quad (7.13)
\]

Moreover, the maximum $P_M$ has to be defined in order to design the operation range of the motor. In this case, 50% is defined as the maximum duty cycle. Therefore, the input saturation is set as in Fig. 7.5.

Subsequently, the DFC input calculation process must be added into the internal loop as shown in Fig. 7.6 and 7.7, respectively.
Fig. 7.5: Relation between $P_M$ and the resultant voltage vector

\[ \alpha_k = -\arctan \left( \frac{V_q}{V_d} \right) \]

\[ V = \sqrt{V_d^2 + V_q^2} \]

Fig. 7.6: DFC input calculation

Fig. 7.7: Internal loop
7.1.2 Current Control Loop

After setting up the internal loop, the central loop is the next step. The current control loop consists of two parts, i.e. $I_q$ and $I_d$ loops. In the order to find the control parameters, the time constant of each loop has to be found.

According to the differential equation in (7.1), $I_q$ and $I_d$ parts can be rewritten in (7.14). It can be figured out that both equations are similar to a $RL$ circuit, when the rotor frequency is zero as represented in (7.15).

$$\frac{dI_d}{dt} = -\frac{R_d I_d}{L_d} + \frac{p_r \omega R L_i I_d}{L_d} + \frac{V_d}{L_d}$$

$$\frac{dI_q}{dt} = -\frac{\rho R L_i I_d}{L_q} - \frac{R_q I_d}{L_q} + \frac{p_r \omega R \Psi_r}{L_q} + \frac{V_q}{L_q}$$

(7.14)

$$\frac{dI_d}{dt} = -\frac{R_d I_d}{L_d} + \frac{V_d}{L_d} \rightarrow V_d = R_d I_d + L_d \frac{dI_d}{dt}$$

$$\frac{dI_q}{dt} = -\frac{R_q I_d}{L_q} + \frac{V_q}{L_q} \rightarrow V_q = R_q I_d + L_q \frac{dI_q}{dt}$$

(7.15)

Basically, the current of the $RL$ circuit is in (7.16) and the time constant ($\tau$) can be found when the current magnitude is at 0.63 times of the saturation current in (7.17).

$$i(t) = i(t)(1 - e^{-\frac{R}{L} t})$$

(7.16)

$$i(t) = i(t)(1 - e^{-t}) = 0.63i(t) : t = \frac{L}{R} = \tau$$

(7.17)

$$i(t) = i(t)(1 - e^{-\infty}) = i(t) : t \rightarrow \infty$$

The $RL$ circuit transfer function in the Laplace domain is also in (7.18). $\tau$ in (7.17) can be added into the equation and the rest terms can be found by calculating the ratio between the measured current and the applied voltage at the saturation level.

For the controller, a PI controller ($G(s)$) is selected. This is because the time constant of the machine is usually small, then the system does not need to accelerate by combining a D part. The PI controller can roughly be evaluated in (7.19) by using the value as a rule of thumb in (7.20) [52–53].
\[
\frac{I(s)}{V(s)} = \frac{1}{R + sL} = \frac{1}{R} \frac{1 - \frac{L}{R} s}{1 + \frac{L}{R} s} = \frac{1}{R} \frac{1}{1 + s\tau} \quad : \quad t \to \infty
\]

\[
G(s) = K_c \frac{1 + \tau s}{s}
\]

\[
K_c = \frac{R}{2\tau}
\]

Regarding the concepts of the \( RL \) circuit, they can be applied to find the time constant, the transfer function and the PI controller of each axis in \((d, q)\) frame.

Therefore, the experimental environment to find the current loop control parameters can be illustrated in Fig. 7.8. The constant voltage vector is applied to each axis and the current of the applied axis is measured. The rotor must be locked in order to force the system to have the same behavior as the \( RL \) circuit in (7.15), while applying the step input as the constant voltage vector. All information of each axis is shown and evaluated.

Fig. 7.8: Experimental environment for current loop control
7.1.2.1 $I_q$ Control Loop

The rotor of PMSM4 is locked. The step input ($V_q$) at 14.24 V is applied. $I_q$ is captured as displayed in Fig. 7.9. Consequently, the time constant in $q$ axis ($\tau_q$) is 3 ms.

![Fig. 7.9: $I_q$ step response](image)

The transfer function and the PI controller of the $q$ axis can be computed in (7.21) and (7.22), respectively.

\[
I \to \infty: \quad \frac{1}{R} = \frac{I_q(s)}{V_q(s)} = \frac{13.061 A}{14.24 V} = 0.917 \quad \Omega^{-1}
\]

\[
H_q(s) = \frac{I_q(s)}{V_q(s)} = \frac{0.917}{1 + 0.003 s} \quad \Omega^{-1}
\]

\[
G_q(s) = 181 \frac{1 + 0.003 s}{s} \quad \Omega
\]
7.1.2.2 $I_d$ Control Loop

The step response of $I_d$ has been done in the same way by applying the $V_d$ step input at 14.24 V. $I_d$ is collected as shown in Fig. 7.10. Hence, the time constant in d axis ($\tau_d$) is 2.85 ms. $\tau_d$ is less than $\tau_q$, which conforms to the relation between $L_d$ and $L_q$ of PMSM ($L_d < L_q$). The transfer function and the PI controller of the $d$ axis are in (7.23) and (7.24).

\[ t \to \infty: \quad \frac{1}{R} = \frac{I_d(s)}{V_d(s)} = \frac{11.682A}{14.24V} = 0.82 \quad \Omega^{-1} \]

\[ H_d(s) = \frac{I_d(s)}{V_d(s)} = \frac{0.82}{1 + 0.00285s \cdot s} \quad \Omega^{-1} \]

\[ G_d(s) = 213.95 \frac{1 + 0.00285s \cdot s}{s} \quad \Omega \]

7.1.2.3 Current Loop Experimental Results

The structure of the central loop is depicted in Fig. 7.11. The DFC structure (internal loop) is combined within this loop. Three experiments are performed by varying the
desired currents in \((d, q)\) axis, the varied desired currents \((I_{d,s}, I_{q,s})\) in Fig. 7.12, 7.14 and 7.16 and the measured currents \((I_{d,m}, I_{q,m})\) of each experiment are illustrated in Fig. 7.13, 7.15 and 7.17, respectively.

Fig. 7.11: Central loop

Fig. 7.12: \(I_{q,s}\) step input
Fig. 7.13: $I_{q,m}$ and $I_{d,m}$ of $I_{q,s}$ step input

Fig. 7.14: $I_{d,s}$ step input
Fig. 7.15: $I_{q,m}$ and $I_{d,m}$ of $I_{d,s}$ step input

Fig. 7.16: $I_{q,s}$ and $I_{d,s}$ step inputs
The measured currents in all cases show that they are under control by the desired values. Consequently, the central loop is achieved.

7.1.3  Closed Loop Speed Control Structure

The closed loop speed control is the external loop, which is the central loop and the internal loop are inside this loop. The speed calculation for the motor mechanical speed \((N_m)\) has to be added. The speed calculation can be done in (7.25).

\[
N_m = \frac{\alpha_{\text{cal}(n)} - \alpha_{\text{cal}(n-1)}}{2\pi p_r T_s}
\]  

(7.25)

Where \(T_s\) is the sampling time for processing \(V_{\text{NAN}}\), which is the sampling time of the modified measuring sequences \((T_s=0.0003\ \text{s})\) and \(n\) is the data sequence. The speed calculation is added into the central loop as shown in Fig. 7.18.

After adding the speed calculation, the step input of \(I_{q,s}\) at 5 A and \(I_{d,s}\) at zero are applied. \(I_{q,m}, I_{d,m}\) are depicted in Fig. 7.19 and \(N_m\) is shown in Fig. 7.20.
Sensorless Closed Loop Speed Control with DFC

Fig. 7.18: Adding speed calculation to central loop

Fig. 7.19: $I_{q,m}$ and $I_{d,m}$ ($I_{q,s}$ at 5 A)
Fig. 7.20: Calculated mechanical rotor speed ($N_m$)

It is noteworthy that the results in Fig. 7.13 and 7.19 are almost the same, except $I_{d,m}$ in Fig. 7.19 is slightly bigger. Moreover, $N_m$ in Fig. 7.20 has also a lot of noise. This is because the noise is available in the measurement parts and when the signals are gained in the calculation, the noise is also increased. However, it does not influence through the system.

Regarding the rotor speed in Fig. 7.20, the time constant for the speed response ($\tau_s$) is 0.76 s. The transfer function and the controller can be estimated in (7.26) and (7.27).

\[
 t \rightarrow \infty: \quad \frac{N_m(s)}{I_{q,m}(s)} = \frac{930}{60s \cdot 5A} = 3.1 \quad \text{A}^{-1}\text{s}^{-1} \\
 H_s(s) = \frac{N_m(s)}{I_{q,m}(s)} = \frac{3.1}{1+0.76s \cdot s} \quad \text{A}^{-1}\text{s}^{-1} \\
 G_s(s) = 0.2122 \frac{1+0.76s \cdot s}{s} \quad \text{As} \\
\]

(7.26)  

(7.27)

Besides, the moment of inertia ($J$) can also be found by using the information in Fig. 7.20. This is because the slope of Fig. 7.20 is the acceleration, where the relation is stated in (7.28).
\[
d\omega_r = \frac{p_r (L_d - L_q) I_d I_d + p_r \Psi_q I_q - T_L}{J}
\]

(7.28)

The experiment has been performed when \( I_d \) and \( T_L \) are zero. Consequently, the relation becomes in (7.29). \( J \) is estimated in (7.30).

\[
d\omega_r = \frac{p_r \Psi_q I_q}{J}
\]

(7.29)

\[
J = \frac{p_r \Psi_q I_q}{2\pi} = \frac{11 \cdot 0.00624 \text{Vs} \cdot 5 \text{A}}{60 \text{s} \cdot 0.76 \text{s}} = 0.0043 \text{ kgm}^2
\]

(7.30)

Subsequently, the electrical torque \((T_m)\) for the ideal case at the maximum value can be calculated in (7.31), where the designed maximum \( I_q \) is 9.69 A.

\[
T_m = p_r \Psi_q I_q = 11 \cdot 0.00624 \text{Vs} \cdot 9.69 \text{A} = 0.67 \text{ Nm}
\]

(7.31)

### 7.1.3.1 Closed Loop Speed Control Tuning

After having all control parameters, the external loop structure with other loops is represented in Fig. 7.21. Before implementing the closed loop control, the uncertainty of the calculated rotor position has to be removed, which can be done as described in chapter 5. The sign of \( I_q \) and \( N_m \) must be the same sign. In this case, the clockwise direction is set as the positive sign.

For instance, \( P_M \) at 12.5 % is applied to drive the motor after removing the uncertainty and setup all signs for the calculation and the control loop is not closed. The required \( \alpha_{cal} \), \( I_{d,m} \), \( I_{q,m} \) and \( N_m \) characteristics are shown in Fig. 7.22. The reason to employ the small \( P_M \) is to generate the slow frequencies, and then the phases of voltage and current are not much different.
Fig. 7.21: Closed loop speed sensorless control with DFC

Fig. 7.22: Required characteristics for closed loop control
Fig. 7.22 shows that $I_{q,m}$ oscillates periodically. Then, $I_{q,m}$ and $\alpha_{cal}$ are zoomed in and displayed in Fig. 7.23. The oscillation frequency is 11 periods of electrical positions, which is one mechanical frequency. It confirms that PMSM4 is the unbalanced motor with the mechanical or structure unbalance.

After closing the loop, the desired speed ($N_s$) is given as the step input and the PI controllers for speed loop and $I_q$ have been tuned. Finally, the proper PI controllers are listed in (7.32). The PI controller for the $I_d$ loop has not been changed, this is because $I_{d,s}$ is set to zero and is not linked to the external loop.

$$G_q(s) = 36 \frac{1 + 0.00136s}{s} \Omega$$

$$G_d(s) = 213.95 \frac{1 + 0.00285s}{s} \Omega \quad (7.32)$$

$$G_s(s) = 18 \frac{1 + 0.9s}{s} \text{ As}$$

The found controllers in (7.32) are used to implement the sensorless closed loop speed control. The experimental results and analysis are given in the next part.
7.2 Experimental Results and Analysis

According to the experiments, there are three experiments, which have been done in order to perform the capabilities of the DFC method to work with the sensorless closed loop speed control, especially for all speeds.

7.2.1 Flip Rotor Direction Test

In this experiment, the desired mechanical speed \((N_s)\) pattern is depicted in Fig. 7.24. The desired speed is changed every two seconds in inverse direction at 600 rpm. The results, i.e. \(I_{q,m}\), \(I_{d,m}\) and \(N_m\) are also in the same figure.

![Fig. 7.24: Flipping rotor direction closed loop speed control experimental results](image)

7.2.2 Stopped Rotor Test

The \(N_s\) sequences have been modified to 600, 0, and \(-600\) rpm, which is always changed in two seconds as shown in Fig. 7.25 including the results.
Fig. 7.25: Stopped rotor closed loop speed control results

Fig. 7.26: Zoomed stopped rotor closed loop speed control results
Moreover, there are three more values, i.e. \( \alpha_{\text{cal}} \), \( \alpha_k \), and \( P_M \), that have been captured while the desired speed is set to zero as illustrated in Fig. 7.26.

### 7.2.3 Applying Load Test

Regarding the test bench structure in Fig. 6.15, the load can be applied by driving the Stromag motor in the opposite direction of the desired speed with the current control mode. The torque load \( (T_L) \) by the Stromag motor is set to 0.1 Nm and applied to the system while PMSM4 is controlled by the sensorless closed loop speed control. Two experiments to apply \( T_L \) have been achieved. Firstly, \( T_L \) is applied, while \( N_s \) is zero or at standstill. Another experiment is done by applying \( T_L \), when the machine is running with \( N_s \) at 600 rpm.

The experimental results of applying load test at different speeds are shown in Fig. 7.27 and 7.28, respectively.

![Graph showing experimental results of applying load test at different speeds.](image)

**Fig. 7.27:** Applied \( T_L \) at standstill \( (N_s=0 \text{ rpm}) \)
Fig. 7.28: Applied $T_L$ while driving the machine ($N_s=600$ rpm)

7.2.4 Summary and Analysis

Regarding the experimental results, the sensorless closed loop speed control with DFC can properly work for all speeds including at standstill as shown in Fig. 7.24 and 7.25. Moreover, the control scheme with DFC can also deal with load as illustrated in Fig. 7.27 and 7.28.

Previously in chapter 3, the calculated electrical rotor position slightly swings at standstill as shown in Fig. 3.21 and 3.24. By applying the sensorless closed loop speed control as a control strategy, the mentioned characteristic has been eliminated. Although the influence of the stator flux through the resultant flux linkage as described in chapter 4 has not been decoupled, the FOC approach can be used to perform the closed loop control by using the DFC position as the exact electrical rotor position. It means that the influence of the PMSM4 stator currents on the rotor flux is not vast, therefore the stator flux does not need to be decoupled from the resultant flux linkage.
Moreover, $I_{q,m}$ in Fig. 7.27 and 7.28 are increased after applying the load in order to keep the motor speed ($N_m$) to be the same as the desired speed ($N_s$). The mentioned compensation strategy is clearly shown in Fig. 7.28 that $N_m$ drops when $T_L$ is applied. Then, $N_m$ is recovered after $I_{q,m}$ is increased by the FOC approach in order to deal with $T_L$.

However, there are two more aspects to consider. Firstly, $I_{q,m}$ is not in fact linear to the torque. This is because the motor is an unbalanced motor and the system tries to recompense in order to keep the constant speed. The compensation process can be also found by taking $P_M$ in Fig. 7.26 into account. Although $I_{q,m}$ in the experiments are not really linear, they are much more linear than $I_{q,m}$ in Fig. 7.23.

![Fig. 7.29: Spectrum of $I_{q,m}$ at $N_m$ 600 rpm](image)

Consequently, $I_{q,m}$ in Fig. 7.25, when $N_m$ is close to $N_s$ (from 1 s to 1.8 s), is converted into the frequency domain. The spectrum of $I_{q,m}$ is represented in Fig. 7.29, the dominant frequency is the DC component. Indeed, it can be concluded that $I_{q,m}$ and $I_{d,m}$ of the closed loop control are DC signals.

Finally, the noise can strongly influence to the closed loop system. In this implementation, the noise comes from two sources, i.e. hardware part and calculation process. The noise from the hardware part can be generated by the measurement circuits e.g. EMI from the inverter. The noise from the calculation can be produced by
using the noisy signals to process including the least significant bit (LSB) of the ADC. For example, $I_{q,m}$ in Fig. 7.25 is increased for a short time, when $N_s$ is zero. This is because $N_m$ is slightly dropped. Thus, $P_M$ is also faintly increased, which influences through $\alpha_{cal}$ for a very short time. In this case, the noise is mainly from the current measurements and the speed calculation.
8 Conclusion and Future Work

8.1 Conclusion

The sensorless rotor position detection technique named Direct Flux Control is proposed in this thesis. The PMSMs neutral point must be accessible to implement DFC. The DFC method is validated and implemented in both software and hardware environments. Four PMSMs are investigated.

In software simulation, the co-simulation between Maxwell and Simulor is selected. This is because the DFC method requires the machine model in a three phase frame with all properties e.g. the nonlinear BH curve, which can be achieved in Maxwell by finite element methods (FEM). The machine model is also validated to confirm that it can act in the same way as the real PMSM. Afterwards, the DFC method is executed by experimenting with the PMSM model. The results show that the flux linkage signals and the calculated electrical rotor position can be obtained accurately, although there are some errors of the calculated electrical rotor position. This is because the stator currents influence the calculated position and other factors of the model are not decoupled and compensated. Moreover, the PMSM structure is analyzed by modifying the number of stator teeth in a software simulation environment. The DFC signals are in better conditions, when the machine structure is asymmetry in order to distribute the inductance and the least common multiple value of the number of stator teeth and rotor magnets are higher to increase the cogging torque frequency.

Additionally, the DFC method can work for a wide range of speed, including standstill as examined in the hardware implementation. At standstill without load, the very small duty cycle pulses, which cannot influence the motor, are applied to measure the voltage signals. Due to less machine saliencies in non-salient poles PMSM, a $d$-component is added to generate the saturation by applying the correction angle.

Consequently, the derivation has been done in order to observe the approximated function of the flux linkage signal, which can be used to improve the capability of DFC. The flux linkage signal approximation is proposed by using the relation of the phase inductances in the stator frame with the DFC conditions. The measuring
sequences are also modified to remove the offset values, which can lead to have unbalanced characteristics of the flux linkage signals. The results also confirm that the DFC method is appropriate for the machine, whose inductances in the synchronous frame are different to create the machine saliencies. Otherwise, the used technique of non-salient poles PMSM has to be applied to change the inductances characteristics to generate the flux linkage signals. The approximated flux linkage signals exhibit the same features as the extracted flux linkage signals from the machines.

The uncertainty of magnet poles has been considered by using the magnetic reluctance characteristics dealing with the flux linkage signal approximation function. Consequently, a technique to remove the uncertainty has been proposed. The machine can be always driven in the correct direction by applying the approach.

Furthermore, four electrical rotor position calculation methods have been researched and developed for DFC. Three methods are based on the relation of flux linkage signals and another one is achieved by modifying the flux linkage signals to be the position signals based on the relation of the phase inductances. The proposed method by using the relation of phase inductances can automatically remove the fourth harmonic from the flux linkage signals without any filtering. The proper adding offset value strategy for the rotor position estimation is also stated. Even though the electrical rotor position calculation by using phase inductances signals is the most accurate estimation method, the method by using the trigonometric relation with flux linkage signals presents almost the same results. This is because the errors are quite small, especially when the magnitude of the inductance ratio is much bigger.

Two highest accuracy rotor estimation methods are examined by evaluating with the measured rotor position. The outcomes show that both estimation techniques give the exact rotor position, which can be used for further applications.

Subsequently, the closed loop speed sensorless control with DFC is designed and implemented. The closed loop speed control can work for all speeds and at standstill, including with and without load. The slightly oscillating flux linkage signals characteristic at standstill is also removed. It means that the DFC method can correctly work, even though the experimented machine is an unbalanced machine. The three phase unbalanced machine leads to have the fundamentally frequency
Conclusion and Future Work

coupling into the flux linkage signals. This is because the inconstant resultant current vector is generated by the asymmetry of the three phase currents, which directly affects through the fluxes and also the inductances in the synchronous frame.

All in all, the DFC method is a continuously working sensorless rotor position estimation method. The machine information e.g. machine parameters are not required. A pre- or self commissioning process of the machine can be foregone. There is only the measuring time constant, which is needed. The different voltage between the neutral points, which is used to extract the flux linkage signals, must be measured only when the switched phase current is stable and the switched DC link voltage is less oscillating.

At last, the experimental results show that the machine itself is used as the sensor to give information such as magnetization state, which are much more than the information given by mechanical sensors. The key of the DFC method, which can work for all speeds and at standstill, is the flux linkage signal. All described properties of the flux linkage signal can be applied to increase the efficiency of the sensorless method, which can also bring many advantages for industrial aspects e.g. the use of materials for proper machine designs and reducing cost by using the DFC method instead of mechanical sensors. Therefore, the DFC method is one of the most recent technologies and also one of the best of the sensorless rotor position estimation techniques.

8.2 Future Work

According to the dissertation achievements, there are several aspects based on the DFC method and the found flux linkage signal approximation function, which are attractive to develop.

8.2.1 Position Calculation

Usually, three flux linkage signals are used to calculate a position, which require four values of $V_{NAN}$ in one original measuring sequence or six values of $V_{NAN}$ in one modified measuring sequence.
Since the approximation function has been found, less than three flux linkage signals should be sufficient to estimate the electrical position. Then, the required numbers of $V_{NAN}$ can be decreased. For instance, the rotor position can be calculated by using two flux linkage signals in (8.1) and only one flux linkage signal in (8.2). However, the inductance ratio ($\xi$) value is required in this case.

$$
\alpha_{cal,PF} = \frac{1}{2} \arcsin((w-v) \cdot \frac{3\xi^2-0.75}{\sqrt{3}\xi V_{DC}})
$$

$$
= \frac{1}{2} \arcsin(\frac{\xi V_{DC} (\cos(2\alpha - \frac{\pi}{3}) - \cos(2\alpha + \frac{\pi}{3}))}{3\xi^2-0.75} \cdot \frac{3\xi^2-0.75}{\sqrt{3}\xi V_{DC}}) \tag{8.1}
$$

$$
= \frac{1}{2} \arcsin(\sin(2\alpha))
$$

$$
= \alpha
$$

$$
\alpha_{cal,PF} = \frac{1}{2} \arccos(-u \cdot \frac{3\xi^2-0.75}{\xi V_{DC}})
$$

$$
= \frac{1}{2} \arccos(\frac{\xi V_{DC} \cos(2\alpha)}{3\xi^2-0.75} \cdot \frac{3\xi^2-0.75}{\xi V_{DC}}) \tag{8.2}
$$

$$
= \frac{1}{2} \arccos(\cos(2\alpha))
$$

$$
= \alpha
$$

8.2.2 Speed Calculation

The rotor speed is directly calculated by using the difference between the estimated electrical rotor positions, which can lead to have noise in the speed signal. Consequently, the speed calculation method should be improved by combining any strategy to decrease the influence of the noise e.g. a recursive filter. One of the well known recursive filters is the Kalman filter, which can also perform in the real time implementation with the probabilistic solution to eliminate the noise of the signals.

8.2.3 Removing Uncertainty

Although the technique to remove the uncertainty has been proposed and implemented, it should be developed by adapting the used characteristics with other ways e.g. the current characteristics. Otherwise, a better way should be found in order
to be easier in calculation. For example, computational intelligence techniques with features extraction might be applied to invent a new method.

### 8.2.4 Decoupling Algorithm

The sensorless closed loop speed control is implemented by using the FOC approach without decoupling the influence of the stator currents. However, this strategy cannot be applied for all motors. This is because several motors have a strong influence of the stator currents on the resultant flux linkage.

As analyzed in Chapter 4, the strong influence of the stator fluxes can be redisplayed in Fig. 8.1. If the magnitude of $\Psi_s$ is not a small value, the resultant flux linkage ($\Psi_p$) will not be narrowly aligned on the rotor flux ($\Psi_r$) vector. The relation between three fluxes is rewritten in (8.3).

$$\Psi_p = \Psi_s + \Psi_r$$  \hspace{1cm} (8.3)

![Fig. 8.1: Flux relations in stationary frame](image)

Generally, the exact electrical rotor position is needed for the control loop. A decoupling algorithm can be done by indirect estimating the stator flux using the stator currents and calculating the electrical rotor angle with the trigonometric relation. Therefore, the decoupling algorithm can be applied to decouple the fluxes for the motors, which have a strong influence of the stator currents, in order to obtain the correct electrical rotor position by DFC before integrating with control strategies.
8.2.5 DFC Applications

Currently, the extracted flux linkage signals are merely used to calculate the electrical rotor position. Actually, the signals can be adapted to use in other fields, e.g. machine fault recognition by considering the inductances characteristics.

For instance, the PMSM4 flux linkage signals spectrums are only the second and the fourth harmonics, but also the first harmonic. It can be implied that the PMSM4 is an unbalanced motor.

Moreover, the magnetization state can be figured out by the DFC method. Whenever the characteristics of the DFC signals are changed, it means that any improper circumstance inside the machine occurs. Thus, the DFC usage in other applications is also interesting to implement.
9 References


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10 Appendix

10.1 Fluxes Conversion from Stationary Frame to Stator Frame

\( \Psi_{s,\alpha} \) and \( \Psi_{s,\beta} \) can be calculated as in (10.1), where \( \Psi_{s,d} \) and \( \Psi_{s,q} \) are in (10.2).

\[
\begin{bmatrix}
\Psi_{s,\alpha} \\
\Psi_{s,\beta}
\end{bmatrix} =
\begin{bmatrix}
\sin(\alpha) & \cos(\alpha) \\
\cos(\alpha) & -\sin(\alpha)
\end{bmatrix}
\begin{bmatrix}
\Psi_{s,d} \\
\Psi_{s,q}
\end{bmatrix}
\]  

\( \Psi_{s,d} \) and \( \Psi_{s,q} \) can be found in (10.3) and (10.4).

\[
\begin{align*}
\Psi_{s,\alpha} &= \frac{L_d}{\sqrt{6}} [(-\sqrt{3}I_V + \sqrt{3}I_W) \sin(\alpha) + (2I_U - I_V - I_W) \cos(\alpha)] + \\
&+ \frac{L_q}{\sqrt{6}} [(-\sqrt{3}I_V + \sqrt{3}I_W) \cos(\alpha) - (2I_U - I_V - I_W) \sin(\alpha)] \cos(\alpha) \\
&= \frac{I_U}{\sqrt{6}} (2L_d \cos(\alpha) \sin(\alpha) - 2L_q \sin(\alpha) \cos(\alpha)) + \quad (10.3) \\
&+ \frac{I_V}{\sqrt{6}} (-L_d \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin^2(\alpha) - \sqrt{3}L_q \cos^2(\alpha) + L_q \sin(\alpha) \cos(\alpha)) + \\
&+ \frac{I_W}{\sqrt{6}} (-L_d \sin(\alpha) \cos(\alpha) + \sqrt{3}L_d \sin^2(\alpha) + \sqrt{3}L_q \cos^2(\alpha) + L_q \sin(\alpha) \cos(\alpha))
\end{align*}
\]

\[
\begin{align*}
\Psi_{s,\beta} &= \frac{L_d}{\sqrt{6}} [(2I_U - I_V - I_W) \cos(\alpha) + (-\sqrt{3}I_V + \sqrt{3}I_W) \sin(\alpha)] \cos(\alpha) - \\
&- \frac{L_q}{\sqrt{6}} [(-\sqrt{3}I_V + \sqrt{3}I_W) \cos(\alpha) - (2I_U - I_V - I_W) \sin(\alpha)] \sin(\alpha) \\
&= \frac{I_U}{\sqrt{6}} (2L_d \cos^2(\alpha) + 2L_q \sin^2(\alpha)) + \quad (10.4) \\
&+ \frac{I_V}{\sqrt{6}} (-L_d \cos^2(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha)) + \\
&+ \frac{I_W}{\sqrt{6}} (-L_d \cos^2(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha))
\end{align*}
\]
After obtaining $\Psi_{s,\alpha}$ and $\Psi_{s,\beta}$, $\Psi_{s,U}$, $\Psi_{s,V}$ and $\Psi_{s,W}$ can be computed in (10.5), and the inductances can be found based on the relation between the current and the inductance as in (10.6).

$$
\begin{bmatrix}
\Psi_{s,U} \\
\Psi_{s,V} \\
\Psi_{s,W}
\end{bmatrix}
=egin{bmatrix}
0 & \frac{\sqrt{2}}{\sqrt{3}} \\
-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}}
\end{bmatrix}
\begin{bmatrix}
\Psi_{s,\alpha} \\
\Psi_{s,\beta}
\end{bmatrix}
\tag{10.5}
$$

$$
\begin{bmatrix}
\Psi_{s,U} \\
\Psi_{s,V} \\
\Psi_{s,W}
\end{bmatrix}
=egin{bmatrix}
L_U & L_{UV} & L_{UW} \\
L_{AV} & L_V & L_{VW} \\
L_{WU} & L_{VW} & L_W
\end{bmatrix}
\begin{bmatrix}
I_U \\
I_V \\
I_W
\end{bmatrix}
\tag{10.6}
$$

Therefore, $\Psi_{s,U}$ can be calculated as following:

$$
\Psi_{s,U} = \sqrt{\frac{2}{3}} \Psi_{s,\beta} = I_U L_U + I_V L_{UV} + I_W L_{UW}
$$

$$
\begin{align*}
&= I_U \left( \frac{1}{3} \left( 2L_d \cos^2(\alpha) + 2L_q \sin^2(\alpha) \right) \right) + \\
&\quad I_V \left( \frac{1}{3} \left( -L_q \cos^2(\alpha) - \sqrt{3}L_d \sin(\alpha) \cos(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha) \right) \right) + \\
&\quad I_W \left( \frac{1}{3} \left( -L_d \cos^2(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha) \right) \right)
\end{align*}
\tag{10.7}
$$

$I_U$, $I_V$ and $I_W$ can be separated as shown in (10.7), consequently $L_U$, $L_{UV}$ and $L_{UW}$ can be calculated as in (10.8) to (10.10), respectively.

$$
L_U = \frac{1}{3} \left( 2L_d \cos^2(\alpha) + 2L_q \sin^2(\alpha) \right)
$$

$$
= \frac{1}{3} \left( 2L_d \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) + 2L_q \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) \right)
\tag{10.8}
$$

$$
= \frac{1}{3} \left( (L_d + L_q) + (L_d - L_q) \cos(2\alpha) \right)
$$
\[ L_{UV} = \frac{1}{3}(-L_d \cos^2(\alpha) - \sqrt{3}L_d \sin(\alpha) \cos(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha)) \]

\[ = \frac{1}{3}\left(-\frac{\sqrt{3}}{2}L_d \sin(2\alpha) + \frac{\sqrt{3}}{2}L_q \sin(2\alpha) - L_d \cos^2(\alpha) - L_q \sin^2(\alpha)\right) \]

\[ = \frac{1}{3}\left((-L_d - L_q)\frac{\sqrt{3}}{2}\sin(2\alpha) - L_d \left(\frac{1}{2} + \frac{1}{2}\cos(2\alpha)\right) - L_q \left(\frac{1}{2} - \frac{1}{2}\cos(2\alpha)\right)\right) \]

\[ = \frac{1}{3}\left(-\frac{1}{2}(L_d + L_q) + (L_d - L_q)(-\frac{\sqrt{3}}{2}\sin(2\alpha) - \frac{1}{2}\cos(2\alpha))\right) \]

\[ = \frac{1}{3}\left(-\frac{1}{2}(L_d + L_q) + (L_d - L_q)(\cos(2\alpha + \frac{\pi}{3}))\right) \]

\[ L_{VW} = \frac{1}{3}(-L_d \cos^2(\alpha) + \sqrt{3}L_d \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha)) \]

\[ = \frac{1}{3}\left(\frac{\sqrt{3}}{2}L_d \sin(2\alpha) - \frac{\sqrt{3}}{2}L_q \sin(2\alpha) - L_d \cos^2(\alpha) - L_q \sin^2(\alpha)\right) \]

\[ = \frac{1}{3}\left((L_d - L_q)\frac{\sqrt{3}}{2}\sin(2\alpha) - L_d \left(\frac{1}{2} + \frac{1}{2}\cos(2\alpha)\right) - L_q \left(\frac{1}{2} - \frac{1}{2}\cos(2\alpha)\right)\right) \]

\[ = \frac{1}{3}\left(-\frac{1}{2}(L_d + L_q) + (L_d - L_q)(\frac{\sqrt{3}}{2}\sin(2\alpha) - \frac{1}{2}\cos(2\alpha))\right) \]

\[ = \frac{1}{3}\left(-\frac{1}{2}(L_d + L_q) + (L_d - L_q)(\cos(2\alpha + \frac{4\pi}{3}))\right) \]
Then, $\Psi_{s,V}$ can be also found as following:

$$
\Psi_{s,V} = -\frac{1}{\sqrt{2}} \Psi_{s,\alpha} - \frac{1}{\sqrt{6}} \Psi_{s,\beta} = L_{VU} I_U + L_{V} I_V + L_{VW} I_W
$$

$$
= I_U \left\{ -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} \left( 2L_d \cos^2(\alpha) + 2L_q \sin^2(\alpha) \right) \right) + 
-\frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} \left( 2L_d \cos(\alpha)\sin(\alpha) - 2L_q \sin(\alpha)\cos(\alpha) \right) \right) \right\}
$$

$$
I_V \left\{ -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} \left( -L_d \cos^2(\alpha) - \sqrt{3}L_d \sin(\alpha)\cos(\alpha) \right) \right) + 
\sqrt{3}L_q \sin(\alpha)\cos(\alpha) - L_q \sin^2(\alpha)) \right) \right\}
$$

$$
+ \frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} \left( -L_q \sin(\alpha)\cos(\alpha) + \sqrt{3}L_q \sin^2(\alpha) \right) \right) + 
-\sqrt{3}L_q \cos^2(\alpha) + L_q \sin(\alpha)\cos(\alpha) \right) \right\}
$$

$$
I_W \left\{ -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} \left( -L_d \cos^2(\alpha) + \sqrt{3}L_d \sin(\alpha)\cos(\alpha) \right) \right) - 
\sqrt{3}L_q \sin(\alpha)\cos(\alpha) - L_q \sin^2(\alpha)) \right) \right\}
$$

$$
- \frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} \left( -L_q \sin(\alpha)\cos(\alpha) + \sqrt{3}L_q \sin^2(\alpha) \right) \right) + 
\sqrt{3}L_q \cos^2(\alpha) + L_q \sin(\alpha)\cos(\alpha) \right) \right\}
$$

(10.11)

$L_{VU}$, $L_{VW}$ and $L_{V}$ can be calculated as in (10.12) to (10.14), respectively.
\[ L_{iv} = -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} (2L_d \cos^2(\alpha) + 2L_y \sin^2(\alpha)) \right) \]
\[ - \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (2L_d \cos(\alpha) \sin(\alpha) - 2L_y \sin(\alpha) \cos(\alpha)) \right) \]
\[ = \frac{1}{3} (-L_d \cos^2(\alpha) - L_y \sin^2(\alpha)) \]
\[ + \frac{1}{3} (-\sqrt{3}L_d \cos(\alpha) \sin(\alpha) + \sqrt{3}L_y \sin(\alpha) \cos(\alpha)) \]
\[ = \frac{1}{3} (-L_d \cos^2(\alpha) - L_y \sin^2(\alpha) - \sqrt{3}L_d \cos(\alpha) \sin(\alpha) + \sqrt{3}L_y \sin(\alpha) \cos(\alpha)) \]
\[ = \frac{1}{3} \left( -\left( L_d \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) - L_y \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) - (L_d - L_y) \frac{\sqrt{3}}{2} \sin(2\alpha) \right) \]
\[ = \frac{1}{3} \left( (-1) \left( L_d + L_y \right) - (L_d - L_y) \frac{1}{2} \cos(2\alpha) - (L_d - L_y) \frac{\sqrt{3}}{2} \sin(2\alpha) \right) \]
\[ = \frac{1}{3} \left( (-1) \left( L_d + L_y \right) + (L_d - L_y) \cos(2\alpha) + \frac{2\pi}{3} \right) \]

\[ (10.12) \]

\[ L_{iv} = -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} (-L_d \cos^2(\alpha) + \sqrt{3}L_d \sin(\alpha) \cos(\alpha) - \sqrt{3}L_y \sin(\alpha) \cos(\alpha) - L_y \sin^2(\alpha)) \right) \]
\[ - \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (-L_d \sin(\alpha) \cos(\alpha) + \sqrt{3}L_d \sin^2(\alpha) + \sqrt{3}L_y \cos^2(\alpha) + L_y \sin(\alpha) \cos(\alpha)) \right) \]
\[ = \frac{1}{3} \left( \frac{L_d}{2} \cos^2(\alpha) - \frac{\sqrt{3}}{2} L_d \sin(\alpha) \cos(\alpha) + \frac{\sqrt{3}}{2} L_y \sin(\alpha) \cos(\alpha) + \frac{L_y}{2} \sin^2(\alpha) \right) \]
\[ + \frac{1}{3} \left( \frac{\sqrt{3}}{2} L_d \sin(\alpha) \cos(\alpha) - \frac{3}{2} L_d \sin^2(\alpha) - \frac{3}{2} L_y \cos^2(\alpha) - \frac{\sqrt{3}}{2} L_y \sin(\alpha) \cos(\alpha) \right) \]
\[ = \frac{1}{3} \left( \frac{L_d}{2} \cos^2(\alpha) + \frac{L_y}{2} \sin^2(\alpha) - \frac{3}{2} L_d \sin^2(\alpha) - \frac{3}{2} L_y \cos^2(\alpha) \right) \]
\[ = \frac{1}{3} \left( \frac{L_d}{2} \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) - \frac{3}{2} L_d \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) \right) \]
\[ + \frac{L_y}{2} \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) - \frac{3}{2} L_y \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{L_d}{2} - \frac{L_y}{2} + L_d \cos(2\alpha) - L_y \cos(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{1}{2} \left( L_d + L_y \right) + (L_d - L_y) \cos(2\alpha) \right) \]

\[ (10.13) \]
\[ L_q = -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} (-L_d \cos^2(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha)) \right) \]

\[ -\frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (-L_d \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin^2(\alpha) - \sqrt{3}L_q \cos^2(\alpha) + L_q \sin(\alpha) \cos(\alpha)) \right) \]

\[ = \frac{1}{3} \left( \frac{L_d}{2} \cos^2(\alpha) + \frac{\sqrt{3}}{2} L_q \sin(\alpha) \cos(\alpha) - \frac{\sqrt{3}}{2} L_q \sin(\alpha) \cos(\alpha) + \frac{L_q}{2} \sin^2(\alpha) \right) \]

\[ + \frac{1}{3} \left( \frac{\sqrt{3}}{2} L_d \sin(\alpha) \cos(\alpha) + \frac{3}{2} L_q \sin^2(\alpha) + \frac{3}{2} L_q \cos^2(\alpha) - \frac{\sqrt{3}}{2} L_q \sin(\alpha) \cos(\alpha) \right) \]

\[ = \frac{1}{3} \left( \frac{L_d}{2} \cos^2(\alpha) + \frac{3}{2} L_q \sin^2(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) \right) \]

\[ + \frac{L_q}{2} \sin^2(\alpha) + \frac{3}{2} L_q \cos^2(\alpha) \]

\[ = \frac{1}{3} \left( (L_d - L_q) \frac{\sqrt{3}}{2} \sin(2\alpha) + L_d \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) + \frac{3}{2} L_q \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) \right) \]

\[ + \frac{L_q}{2} \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) + \frac{3}{2} L_q \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) \]

\[ = \frac{1}{3} \left( (L_d - L_q) \frac{\sqrt{3}}{2} \sin(2\alpha) + L_d + L_q + (L_d - L_q) (-\frac{1}{2} \cos(2\alpha)) \right) \]

\[ = \frac{1}{3} \left( (L_d + L_q) + (L_d - L_q) (-\frac{1}{2} \cos(2\alpha) + \frac{\sqrt{3}}{2} \sin(2\alpha)) \right) \]

\[ = \frac{1}{3} \left( (L_d + L_q) + (L_d - L_q) (-\frac{1}{2} \cos(2\alpha) + \frac{\sqrt{3}}{2} \sin(2\alpha)) \right) \]

\[ = \frac{1}{3} \left( (L_d + L_q) + (L_d - L_q) (\cos(2\alpha + \frac{4\pi}{3})) \right) \]

\[ (10.14) \]
Subsequently, the same way can be applied in order to find \( \Psi_{s,W} \) as below:

\[
\Psi_{s,W} = \frac{1}{\sqrt{2}} \Psi_{s,\alpha} - \frac{1}{\sqrt{6}} \Psi_{s,\beta} = L_{WU} I_U + L_{WV} I_V + L_W I_\omega
\]

\[
= I_U \left( -\frac{1}{\sqrt{6}} \left( \frac{1}{6} (2L_d \cos^2(\alpha) + 2L_q \sin^2(\alpha)) \right) + \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (2L_d \cos(\alpha) \sin(\alpha) - 2L_q \sin(\alpha) \cos(\alpha)) \right) \right) +
\]

\[
I_V \left( -\frac{1}{\sqrt{6}} \left( \frac{1}{6} (-L_d \cos^2(\alpha) - \sqrt{3}L_d \sin(\alpha) \cos(\alpha)) \right) + \frac{\sqrt{3}}{\sqrt{6}} L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha) \right) +
\]

\[
I_\omega \left( -\frac{1}{\sqrt{6}} \left( \frac{1}{6} (-L_d \cos^2(\alpha) + \sqrt{3}L_d \sin(\alpha) \cos(\alpha)) \right) - \frac{\sqrt{3}}{\sqrt{6}} L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha) \right) +
\]

\[
+ \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (-L_d \sin(\alpha) \cos(\alpha) + \sqrt{3}L_d \sin^2(\alpha)) \right) + \frac{\sqrt{3}}{\sqrt{6}} L_q \cos^2(\alpha) \right)
\]

\( L_{WU}, L_{WV} \) and \( L_W \) are stated in (10.16) to (10.18), respectively.
Appendix

\[ L_{uv} = -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} (2L_u \cos^2(\alpha) + 2L_q \sin^2(\alpha)) \right) \]
\[ + \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (2L_u \cos(\alpha) \sin(\alpha) - 2L_q \sin(\alpha) \cos(\alpha)) \right) \]
\[ = -L_u \cos^2(\alpha) - \frac{L_q}{3} \sin^2(\alpha) + \frac{1}{\sqrt{3}} \left( \frac{L_u}{2} \sin(2\alpha) - \frac{L_q}{2} \sin(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{1}{2} (L_u + L_q) - L_u \sin(2\alpha) \cos(\alpha) + \frac{\sqrt{3}}{2} \sin(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{1}{2} (L_u + L_q) + (L_u - L_q) \cos(2\alpha) - \frac{\sqrt{3}}{2} \sin(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{1}{2} (L_u + L_q) + (L_u - L_q) \cos(2\alpha) + \frac{\sqrt{3}}{3} \sin(2\alpha) \right) \]

(10.16)

\[ L_{vv} = -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} (-L_u \cos^2(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) + \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha)) \right) \]
\[ + \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (-L_q \sin(\alpha) \cos(\alpha) - \sqrt{3}L_u \cos^2(\alpha) + \sqrt{3}L_u \sin(\alpha) \cos(\alpha)) \right) \]
\[ = \frac{1}{3} \left( \frac{L_u}{2} \cos^2(\alpha) + L_q \sin(\alpha) \cos(\alpha) - \sqrt{3} \frac{L_q}{2} \sin(\alpha) \cos(\alpha) + \frac{L_u}{2} \sin^2(\alpha) \right) \]
\[ + \frac{1}{3} \left( L_u \sin(\alpha) \cos(\alpha) - \frac{3}{2} L_u \sin^2(\alpha) - \frac{3}{2} \frac{L_q}{2} \cos^2(\alpha) + \frac{\sqrt{3}}{2} L_q \sin(\alpha) \cos(\alpha) \right) \]
\[ = \frac{1}{3} \left( L_u \cos^2(\alpha) - \frac{3}{2} L_u \sin^2(\alpha) + \frac{L_q}{2} \sin^2(\alpha) - \frac{3}{2} \frac{L_q}{2} \cos^2(\alpha) \right) \]
\[ = \frac{1}{3} \left( \frac{L_u}{2} \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) - \frac{L_u}{2} \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) \right) \]
\[ + \frac{L_q}{2} \left( \frac{1}{2} - \frac{1}{2} \cos(2\alpha) \right) - \frac{3}{2} L_q \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{L_u}{2} - \frac{L_q}{2} + L_u \cos(2\alpha) - L_q \cos(2\alpha) \right) \]
\[ = \frac{1}{3} \left( -\frac{1}{2} (L_u + L_q) + (L_u - L_q) \cos(2\alpha) \right) \]

(10.17)
\[ L_w = -\frac{1}{\sqrt{6}} \left( \frac{1}{\sqrt{6}} (-L_d \cos^2(\alpha) + \sqrt{3}L_d \sin(\alpha) \cos(\alpha) - \sqrt{3}L_q \sin(\alpha) \cos(\alpha) - L_q \sin^2(\alpha)) \right) \\
+ \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{6}} (-L_d \sin(\alpha) \cos(\alpha) + \sqrt{3}L_d \sin^2(\alpha) + \sqrt{3}L_q \cos^2(\alpha) + L_q \sin(\alpha) \cos(\alpha)) \right) \\
= \frac{1}{3} \left( \frac{L_d}{2} \cos^2(\alpha) - \frac{\sqrt{3}}{2} L_d \sin(\alpha) \cos(\alpha) + \frac{\sqrt{3}}{2} L_q \sin(\alpha) \cos(\alpha) + \frac{L_d}{2} \sin^2(\alpha) \right) \\
+ \frac{1}{3} \left( -\frac{\sqrt{3}}{2} L_d \sin(\alpha) \cos(\alpha) + \frac{3}{2} L_d \sin^2(\alpha) + \frac{3}{2} L_q \cos^2(\alpha) + \frac{\sqrt{3}}{2} L_q \sin(\alpha) \cos(\alpha) \right) \\
= \frac{1}{3} \left( -\sqrt{3} L_d \sin(\alpha) \cos(\alpha) + \sqrt{3} L_q \sin(\alpha) \cos(\alpha) + \frac{L_d}{2} \cos^2(\alpha) + \frac{3}{2} L_d \sin^2(\alpha) \right) \\
+ \frac{L_q}{2} \sin^2(\alpha) + \frac{3}{2} L_q \cos^2(\alpha)) \\
= \frac{1}{3} \left( -(L_d - L_q) \frac{\sqrt{3}}{2} \sin(2\alpha) + \frac{L_d}{2} (1 + \frac{1}{2} \cos(2\alpha)) + \frac{3L_d}{2} (\frac{1}{2} - \frac{1}{2} \cos(2\alpha)) \right) \\
+ \frac{L_d}{2} (\frac{1}{2} - \frac{1}{2} \cos(2\alpha)) + \frac{3}{2} L_q (\frac{1}{2} + \frac{1}{2} \cos(2\alpha)) \\
= \frac{1}{3} \left( -(L_d - L_q) \frac{\sqrt{3}}{2} \sin(2\alpha) + L_d - \frac{1}{2} L_d \cos(2\alpha) + L_q + \frac{1}{2} L_q \cos(2\alpha) \right) \\
= \frac{1}{3} \left( (L_d + L_q) - (L_d - L_q) \frac{\sqrt{3}}{2} \sin(2\alpha) - (L_d - L_q) \frac{1}{2} L_d \cos(2\alpha) \right) \\
= \frac{1}{3} \left( (L_d + L_q) - (L_d - L_q) \left( \frac{\sqrt{3}}{2} \sin(2\alpha) + \frac{1}{2} \cos(2\alpha) \right) \right) \\
= \frac{1}{3} \left( (L_d + L_q) + (L_d - L_q) \left( -\frac{\sqrt{3}}{2} \sin(2\alpha) - \frac{1}{2} \cos(2\alpha) \right) \right) \\
= \frac{1}{3} \left( (L_d + L_q) + (L_d - L_q) (\cos(2\alpha + \frac{2\pi}{3})) \right) \\
\]

(10.18)
All inductances can be concluded as in (10.19).

\[
L_d = \frac{1}{3} \left( (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha) \right)
\]

\[
L_{uv} = \frac{1}{3} \left( \frac{-1}{2} (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha + \frac{2\pi}{3}) \right)
\]

\[
L_{uw} = \frac{1}{3} \left( \frac{-1}{2} (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha + \frac{4\pi}{3}) \right)
\]

\[
L_{vu} = \frac{1}{3} \left( \frac{-1}{2} (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha + \frac{2\pi}{3}) \right)
\]

\[
L_v = \frac{1}{3} \left( (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha + \frac{4\pi}{3}) \right)
\]

\[
L_{vw} = \frac{1}{3} \left( \frac{-1}{2} (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha) \right)
\]

\[
L_{wu} = \frac{1}{3} \left( \frac{-1}{2} (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha + \frac{4\pi}{3}) \right)
\]

\[
L_{uw} = \frac{1}{3} \left( \frac{-1}{2} (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha) \right)
\]

\[
L_{wv} = \frac{1}{3} \left( (L_{d_d} + L_{q_d}) + (L_{d_d} - L_{q_d}) \cos(2\alpha + \frac{2\pi}{3}) \right)
\]

(10.19)

10.2 Flux Linkage Signal Equation

The flux linkage signals are in (10.20). \( u \), \( v \) and \( w \) can be computed in (10.21) to (10.23), respectively.

\[
u = \frac{\left( L_x + L_y \cos(2\alpha + \frac{2\pi}{3}) \right) \left( L_x + L_y \cos(2\alpha + \frac{4\pi}{3}) \right)}{L_{\text{sum}}^2} V_{DC} - \frac{1}{3} V_{DC}
\]

\[
v = \frac{\left( L_x + L_y \cos(2\alpha + \frac{2\pi}{3}) \right) \left( L_x + L_y \cos(2\alpha) \right)}{L_{\text{sum}}^2} V_{DC} - \frac{1}{3} V_{DC}
\]

\[
w = \frac{\left( L_x + L_y \cos(2\alpha + \frac{2\pi}{3}) \right) \left( L_x + L_y \cos(2\alpha + \frac{4\pi}{3}) \right)}{L_{\text{sum}}^2} V_{DC} - \frac{1}{3} V_{DC}
\]

(10.20)
\[ u = \frac{1}{L^2_{\text{sum}}} (L_x^2 + L_y L_z \cos(2\alpha + \frac{2\pi}{3}) + L_y L_z \cos(2\alpha + \frac{4\pi}{3})) + L_z \cos(2\alpha + \frac{2\pi}{3}) L_z \cos(2\alpha + \frac{4\pi}{3})) V_{dc} - \frac{1}{3} V_{dc} \]

\[ = \frac{1}{L^2_{\text{sum}}} (L_x^2 + L_y L_z \cos(2\alpha + \frac{2\pi}{3}) + \cos(2\alpha + \frac{4\pi}{3})) \]

\[ + L_x^2 \cos(2\alpha + \frac{2\pi}{3}) L_x \cos(2\alpha + \frac{4\pi}{3})) V_{dc} - \frac{1}{3} V_{dc} \]

\[ = V_{dc} \frac{L_x^2}{L^2_{\text{sum}}} + L_y L_z (\cos(2\alpha) \cos(\frac{2\pi}{3}) - \sin(2\alpha) \sin(\frac{2\pi}{3})) + \cos(2\alpha) \cos(\frac{2\pi}{3}) - \sin(2\alpha) \sin(\frac{4\pi}{3})) \]

\[ + L_y^2 \cos(2\alpha) \cos(\frac{2\pi}{3}) - \sin(2\alpha) \sin(\frac{2\pi}{3})) \cos(2\alpha) \cos(\frac{4\pi}{3}) - \sin(2\alpha) \sin(\frac{4\pi}{3})) \]

\[ + \frac{1}{3} V_{dc} \]

\[ = \frac{V_{dc}}{L^2_{\text{sum}}} (L_x^2 + L_y L_z (-0.5 \cos(2\alpha) - \sqrt{3} \sin(2\alpha) - 0.5 \cos(2\alpha) + \sqrt{3} \sin(2\alpha))\]

\[ + L_x^2 (-0.5 \cos(2\alpha) - \sqrt{3} \sin(2\alpha))(-0.5 \cos(2\alpha) + \sqrt{3} \sin(2\alpha)) - \frac{1}{3} V_{dc} \]

\[ = \frac{V_{dc}}{L^2_{\text{sum}}} (L_x^2 - L_x L_y \cos(2\alpha) + L_y^2 ((-0.5 \cos(2\alpha))^2 - (\sqrt{3} \sin(2\alpha))^2)) - \frac{1}{3} V_{dc} \]

\[ = \frac{V_{dc}}{L^2_{\text{sum}}} (L_x^2 - L_x L_y \cos(2\alpha) + L_y^2 (\frac{1}{4} \cos^2(2\alpha) - \frac{3}{4} \sin^2(2\alpha))) - \frac{1}{3} V_{dc} \]

\[ = \frac{V_{dc}}{L^2_{\text{sum}}} (L_x^2 - L_x L_y \cos(2\alpha) + L_y^2 (\cos^2(2\alpha) - \frac{3}{8} + \frac{3}{8} \cos(4\alpha)) - \frac{1}{3} V_{dc} \]

\[ = \frac{V_{dc}}{L^2_{\text{sum}}} (L_x^2 - 0.25L_y^2 - L_x L_y \cos(2\alpha) + 0.5L_y^2 \cos(4\alpha)) - \frac{1}{3} V_{dc} \]

(10.21)
\[ v = \frac{V_{DC}}{L_{sum}} \left( L_1^2 + L_1 L_y \cos(2\alpha + \frac{2\pi}{3}) + L_1 L_y \cos(2\alpha) + L_1^2 \cos(2\alpha + \frac{2\pi}{3}) \cos(2\alpha) \right) - \frac{1}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 + L_1 L_y \cos(2\alpha) \cos(\frac{2\pi}{3}) \sin(2\alpha) \sin(\frac{2\pi}{3}) + L_1 L_y \cos(2\alpha) \right) + \frac{L_1^2 (\cos(2\alpha) \cos(\frac{2\pi}{3}) - \sin(2\alpha) \sin(\frac{2\pi}{3})) \cos(2\alpha))}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 + L_1 L_y (-0.5 \cos(2\alpha) - \frac{\sqrt{3}}{2} \sin(2\alpha) + \cos(2\alpha)) \right) + \frac{L_1^2 (-0.5 \cos(2\alpha) - \frac{\sqrt{3}}{2} \sin(2\alpha) \cos(2\alpha))}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 + L_1 L_y (0.5 \cos(2\alpha) - \frac{\sqrt{3}}{2} \sin(2\alpha)) \right) + \frac{L_1^2 (-0.5 \cos(2\alpha) - \frac{\sqrt{3}}{2} \sin(2\alpha) \cos(2\alpha))}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 + L_1 L_y \cos(2\alpha + \frac{\pi}{3}) + L_1^2 \left( -\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \cos(4\alpha) \right) - \frac{\sqrt{3}}{2} \sin(2\alpha) \cos(2\alpha) \right) \right) - \frac{1}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 + L_1 L_y \cos(2\alpha + \frac{\pi}{3}) + L_1^2 \left( -\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \cos(4\alpha) - \frac{\sqrt{3}}{4} \sin(4\alpha) \right) \right) \right) - \frac{1}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 - 0.25 L_1^2 + L_1 L_y \cos(2\alpha + \frac{\pi}{3}) + 0.5 L_1^2 \left( -\frac{1}{2} \cos(4\alpha) - \frac{\sqrt{3}}{2} \sin(4\alpha) \right) \right) - \frac{1}{3} V_{DC} \]

\[ = \frac{V_{DC}}{L_{sum}^2} \left( L_1^2 - 0.25 L_1^2 + L_1 L_y \cos(2\alpha + \frac{\pi}{3}) + 0.5 L_1^2 \cos(4\alpha + \frac{2\pi}{3}) \right) - \frac{1}{3} V_{DC} \]

(10.22)
\[
\begin{align*}
\text{w} &= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 + L_x L_y \cos(2\alpha + \frac{4\pi}{3}) + L_x L_y \cos(2\alpha) + L_y^2 \cos(2\alpha + \frac{4\pi}{3}) \cos(2\alpha)) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 + L_x L_y \cos(2\alpha) \cos(\frac{4\pi}{3}) - \sin(2\alpha) \sin(\frac{4\pi}{3})) + L_x L_y \cos(2\alpha) \\
&\quad + L_y^2 \left( \cos(2\alpha) \cos(\frac{4\pi}{3}) - \sin(2\alpha) \sin(\frac{4\pi}{3}) \right) \cos(2\alpha)) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 + L_x L_y (-0.5 \cos(2\alpha) + \sqrt{3} \sin(2\alpha)) + \cos(2\alpha)) \\
&\quad + L_y^2 (-0.5 \cos(2\alpha) + \frac{\sqrt{3}}{2} \sin(2\alpha)) \cos(2\alpha)) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 + L_x L_y (0.5 \cos(2\alpha) + \frac{\sqrt{3}}{2} \sin(2\alpha)) \\
&\quad + L_y^2 (-0.5 \cos^2(2\alpha) + \frac{\sqrt{3}}{2} \sin(2\alpha)) \cos(2\alpha)) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 + L_x L_y \cos(2\alpha - \frac{\pi}{3}) + L_x^2 (- \frac{1}{4} + \frac{1}{4} \cos(4\alpha) + \frac{\sqrt{3}}{4} \sin(4\alpha)) \cos(2\alpha)) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 - 0.25L_x^2 + L_x L_y \cos(2\alpha - \frac{\pi}{3}) + 0.5L_x^2 (- \frac{1}{2} \cos(4\alpha) + \frac{\sqrt{3}}{2} \sin(4\alpha)) \cos(2\alpha)) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 - 0.25L_x^2 + L_x L_y \cos(2\alpha - \frac{\pi}{3}) + 0.5L_x^2 \cos(4\alpha - \frac{2\pi}{3})) - \frac{1}{3} V_{DC} \\
&= \frac{V_{DC}}{L_{\text{sum}}^2} (L_x^2 - 0.25L_x^2 + L_x L_y \cos(2\alpha - \frac{\pi}{3}) + 0.5L_x^2 \cos(4\alpha - \frac{2\pi}{3})) - \frac{1}{3} V_{DC}
\end{align*}
\]

(10.23)
10.3 DFC Input Characteristic

The three phase input voltage is firstly stated in (3.21) and rewritten in (10.24).

\[
V_u = \frac{P_m}{100} V_{dc} \sin(\alpha_{cal} + \alpha_k)
\]
\[
V_v = \frac{P_m}{100} V_{dc} \sin(\alpha_{cal} + \alpha_k + \frac{2\pi}{3})
\]
\[
V_w = \frac{P_m}{100} V_{dc} \sin(\alpha_{cal} + \alpha_k - \frac{2\pi}{3})
\]  
(10.24)

\(\alpha_m\) is the summation of \(\alpha_{cal}\) and \(\alpha_k\). Then, the three phase input voltage equation is in (10.25).

\[
\alpha_m = \alpha_{cal} + \alpha_k
\]  
(10.25)

\[
V_u = \frac{P_m}{100} V_{dc} \sin(\alpha_m)
\]
\[
V_v = \frac{P_m}{100} V_{dc} \sin(\alpha_m + \frac{2\pi}{3})
\]  
(10.26)
\[
V_w = \frac{P_m}{100} V_{dc} \sin(\alpha_m - \frac{2\pi}{3})
\]

The three phase voltages can be converted into the stationary frame \((\alpha, \beta)\) by using the same relation in (5.7) as stated in (10.27) and (10.28).

\[
\begin{bmatrix}
V_{\alpha} \\
V_{\beta}
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{\sqrt{2}}{3} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
\end{bmatrix} \begin{bmatrix}
V_u \\
V_v \\
V_w
\end{bmatrix}
\]  
(10.27)

\[
= \begin{bmatrix}
\frac{1}{\sqrt{6}} (-\sqrt{3}V_v + \sqrt{3}V_w) \\
\frac{1}{\sqrt{6}} (2V_u - V_v - V_w)
\end{bmatrix}
\]
\[
\begin{bmatrix}
V_a \\
V_b
\end{bmatrix} = \begin{bmatrix}
\frac{P_M}{100} V_{DC} \frac{1}{\sqrt{2}} (-\sin(\alpha_m + \frac{2\pi}{3}) + \sin(\alpha_m + \frac{4\pi}{3})) \\
\sqrt{\frac{3}{2}} (V_U)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{P_M}{100} V_{DC} \frac{1}{\sqrt{2}} (-(-0.5\sin(\alpha_m) + \frac{\sqrt{3}}{2}\cos(\alpha_m)) + (-0.5\sin(\alpha_m) - \frac{\sqrt{3}}{2}\cos(\alpha_m))) \\
\sqrt{\frac{3}{2}} (V_U)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-\sqrt{\frac{3}{2}} \frac{P_M}{100} V_{DC} \cos(\alpha_m) \\
\sqrt{\frac{3}{2}} \frac{P_M}{100} V_{DC} \sin(\alpha_m)
\end{bmatrix}
\]

\[
(10.28)
\]

The voltages in the stationary frame can be converted to the synchronous frame in (10.29) and (10.30), respectively.

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} = \begin{bmatrix}
\sin(\alpha_{cal}) & \cos(\alpha_{cal}) \\
\cos(\alpha_{cal}) & -\sin(\alpha_{cal})
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b
\end{bmatrix}
\]

\[
(10.29)
\]

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} = \begin{bmatrix}
\sin(\alpha_{cal}) & \cos(\alpha_{cal}) \\
\cos(\alpha_{cal}) & -\sin(\alpha_{cal})
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\sin(\alpha_{cal}) & \cos(\alpha_{cal}) \\
\cos(\alpha_{cal}) & -\sin(\alpha_{cal})
\end{bmatrix} \begin{bmatrix}
-\sqrt{\frac{3}{2}} \frac{P_M}{100} V_{DC} \cos(\alpha_m) \\
\sqrt{\frac{3}{2}} \frac{P_M}{100} V_{DC} \sin(\alpha_m)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{3}{2} \frac{P_M}{100} V_{DC} (-\cos(\alpha_m) \sin(\alpha_{cal}) + \sin(\alpha_m) \cos(\alpha_{cal})) \\
\frac{3}{2} \frac{P_M}{100} V_{DC} (-\cos(\alpha_m) \cos(\alpha_{cal}) - \sin(\alpha_m) \sin(\alpha_{cal}))
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{3}{2} \frac{P_M}{100} V_{DC} \sin(\alpha_m - \alpha_{cal}) \\
-\frac{3}{2} \frac{P_M}{100} V_{DC} \cos(\alpha_m - \alpha_{cal})
\end{bmatrix}
\]

\[
(10.30)
\]

\[\alpha_s\] can be found by using the trigonometric relation as shown in (10.31) and (10.32).
\[ \tan(\alpha_{\text{cal}} - \alpha_m) = \frac{V_d}{V_q} \]
\[ = \frac{\sqrt{3} P_m V_{\text{DC}} \sin(\alpha_m - \alpha_{\text{cal}})}{\sqrt{2} \frac{P_m}{100} V_{\text{DC}} \cos(\alpha_m - \alpha_{\text{cal}})} \]
\[ = -\frac{\sin(\alpha_m - \alpha_{\text{cal}})}{\cos(\alpha_m - \alpha_{\text{cal}})} \]
\[ = \frac{\sin(\alpha_{\text{cal}} - \alpha_m)}{\cos(\alpha_{\text{cal}} - \alpha_m)} \]

\[ -\alpha_k = \alpha_{\text{cal}} - \alpha_m \]
\[ = \arctan\left(\frac{V_d}{V_q}\right) \quad (10.32) \]

While rotating the machine, \( \alpha_k \) is usually set to zero, when the machine saliencies are available. The input voltage vector of the DFC method can be found in (10.33).

\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix}
= \begin{bmatrix}
\frac{\sqrt{3} P_m}{\sqrt{2} \frac{P_m}{100} V_{\text{DC}}} \sin(\alpha_{\text{cal}} + \alpha_k - \alpha_{\text{cal}}) \\
-\frac{\sqrt{3} P_m}{\sqrt{2} \frac{P_m}{100} V_{\text{DC}}} \cos(\alpha_{\text{cal}} + \alpha_k - \alpha_{\text{cal}})
\end{bmatrix}
\]
\[ = \begin{bmatrix}
\frac{\sqrt{3} P_m}{\sqrt{2} \frac{P_m}{100} V_{\text{DC}}} \sin(0) \\
-\frac{\sqrt{3} P_m}{\sqrt{2} \frac{P_m}{100} V_{\text{DC}}} \cos(0)
\end{bmatrix} \quad (10.33) \]
\[ = \begin{bmatrix}
0 \\
-\sqrt{3} \frac{P_m}{\sqrt{2} \frac{P_m}{100} V_{\text{DC}}}
\end{bmatrix} \]

It means that the DFC input voltage vector is directly applied aligned on the \( q \) axis, which leads to have the flux linkage signals with the constant values of \( L_d \) and \( L_q \), which is similar to the field oriented control (FOC) strategy. The difference is only that FOC requires the input current vector aligned on the \( q \) axis in order to generate the maximum torque.
10.4 Unbalanced Motor Current

Due to unbalanced motor phase currents, there are two causes, i.e. driving system and motor itself. The exact cause can be found by measuring the phase currents with two connection types as illustrated in Fig. 10.1 and analyzed as below:

- **Unbalance by Driving System:**
  The characteristic of $I_{U1}$ is similar to $I_{U2}$, as well as $I_{V1}$ and $I_{V2}$, also $I_{W1}$ and $I_{W2}$.

- **Unbalance by Motor Itself:**
  The characteristic of $I_{U1}$ is similar to $I_{W2}$, as well as $I_{V1}$ and $I_{U2}$, also $I_{W1}$ and $I_{V2}$.

![Diagram of PMSM connection types](image)

Fig. 10.1: Three phase connection

Consequently, PMSM4 has been experimented by measuring the phase currents, while the motor is being driven by two connection types at $V_{DC}$ 30 V. The experimental results are in Table 10.1 for the normal connection and in Table 10.2 for the crossed phases connection.

Based on the measured results, the phase currents of the normal connection can be concluded as $I_{U1} > I_{V1} > I_{W1}$. The phase currents of the crossed phases connection trend are $I_{W2} > I_{U2} > I_{V2}$.

Therefore, the measured currents characteristics of both connections conform to the unbalance by motor itself condition. PMSM4 is the unbalanced three phase motor.
### Appendix

#### Table 10.1: Machine phase current ($I_{\text{rms}}$) by normal connection

<table>
<thead>
<tr>
<th>$P_M$ [%]</th>
<th>$I_{U1}$ [A]</th>
<th>$I_{V1}$ [A]</th>
<th>$I_{W1}$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.67</td>
<td>1.3</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>27.78</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
</tr>
<tr>
<td>44.44</td>
<td>3</td>
<td>2.95</td>
<td>2.52</td>
</tr>
<tr>
<td>55.55</td>
<td>4.5</td>
<td>4.5</td>
<td>3.8</td>
</tr>
</tbody>
</table>

#### Table 10.2: Machine phase current ($I_{\text{rms}}$) by crossed phases connection

<table>
<thead>
<tr>
<th>$P_M$ [%]</th>
<th>$I_{U2}$ [A]</th>
<th>$I_{V2}$ [A]</th>
<th>$I_{W2}$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.67</td>
<td>1.3</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>27.78</td>
<td>1.75</td>
<td>1.46</td>
<td>1.75</td>
</tr>
<tr>
<td>44.44</td>
<td>2.86</td>
<td>2.28</td>
<td>2.85</td>
</tr>
<tr>
<td>55.55</td>
<td>4.35</td>
<td>3.55</td>
<td>4.5</td>
</tr>
</tbody>
</table>
Appendix

10.5 Frame Conversion for FOC

The three phase input current in (10.34) is assumed to have the same characteristic as the three phase voltage equation in (10.26).

\[
I_U = I_m \sin(\alpha_m) \\
I_V = I_m \sin(\alpha_m + \frac{2\pi}{3}) \\
I_W = I_m \sin(\alpha_m - \frac{2\pi}{3})
\]  

(10.34)

The three phase currents can be converted into the stationary frame \((\alpha, \beta)\) in (10.35) and (10.36).

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} =
\begin{bmatrix}
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{\sqrt{2}}{3} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}}
\end{bmatrix}
\begin{bmatrix}
I_U \\
I_V \\
I_W
\end{bmatrix}
\]

(10.35)

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} =
\begin{bmatrix}
\frac{\sqrt{3}}{6} (-\sqrt{3}I_V + \sqrt{3}I_W) \\
\frac{1}{\sqrt{6}} (2I_U - I_V - I_W)
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} =
\begin{bmatrix}
\frac{I_m}{\sqrt{2}} (-\sin(\alpha_m + \frac{2\pi}{3}) + \sin(\alpha_m + \frac{4\pi}{3})) \\
\frac{\sqrt{3}}{2} (I_U)
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} =
\begin{bmatrix}
\frac{I_m}{\sqrt{2}} ((-0.5 \sin(\alpha_m) + \frac{\sqrt{3}}{2} \cos(\alpha_m)) + (-0.5 \sin(\alpha_m) - \frac{\sqrt{3}}{2} \cos(\alpha_m))) \\
\frac{\sqrt{3}}{2} I_U
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} =
\begin{bmatrix}
-\frac{3}{2} I_m \cos(\alpha_m) \\
\frac{3}{2} I_m \sin(\alpha_m)
\end{bmatrix}
\]

(10.36)
The currents in the stationary frame have to be converted to the synchronous frame in order to have the dc behavior, which is required for FOC. Therefore, $I_q$ is defined to have positive values while rotating in clockwise direction (viewing from in front of the motor) and negative values while rotating in the counterclockwise direction.

Regarding the conversion matrix in (10.29), the results exhibit in the opposite direction of the defined $I_q$. Thus, the sign of the conversion matrix has to be changed as in (10.37) and calculated in (10.38).

\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} =
\begin{bmatrix}
-\sin(\alpha_{\text{cal}}) & -\cos(\alpha_{\text{cal}}) \\
-\cos(\alpha_{\text{cal}}) & \sin(\alpha_{\text{cal}})
\end{bmatrix}
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix}
\]  
(10.37)

\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} =
\begin{bmatrix}
-\sin(\alpha_{\text{cal}}) & -\cos(\alpha_{\text{cal}}) \\
-\cos(\alpha_{\text{cal}}) & \sin(\alpha_{\text{cal}})
\end{bmatrix}
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix}
= 
\begin{bmatrix}
-\sin(\alpha_{\text{cal}}) & -\cos(\alpha_{\text{cal}}) \\
-\cos(\alpha_{\text{cal}}) & \sin(\alpha_{\text{cal}})
\end{bmatrix}
\begin{bmatrix}
-\frac{3}{2}I_m \cos(\alpha_m) \\
\frac{3}{2}I_m \sin(\alpha_m)
\end{bmatrix}
= 
\begin{bmatrix}
\frac{3}{2}I_m (\cos(\alpha_m) \sin(\alpha_{\text{cal}}) - \sin(\alpha_m) \cos(\alpha_{\text{cal}})) \\
\frac{3}{2}I_m (\cos(\alpha_m) \cos(\alpha_{\text{cal}}) + \sin(\alpha_m) \sin(\alpha_{\text{cal}}))
\end{bmatrix}
\]  
(10.38)

$\alpha_s$ can be found in the same way as calculated for the voltage equation, which is recomputed in (10.39) and (10.40). While rotating the machine in the clockwise direction, $\alpha_s$ is set to zero. The $(d, q)$ current vectors can be found in (10.41).
\[ \tan(\alpha_{cal} - \alpha_m) = \frac{I_d}{I_q} \]
\[ = -\frac{\sqrt{3}}{2} I_M \sin(\alpha_m - \alpha_{cal}) \]
\[ = \frac{\sqrt{3}}{2} I_M \cos(\alpha_m - \alpha_{cal}) \]
\[ = -\frac{\sin(\alpha_m - \alpha_{cal})}{\cos(\alpha_m - \alpha_{cal})} \]
\[ = \frac{\sin(\alpha_{cal} - \alpha_m)}{\cos(\alpha_{cal} - \alpha_m)} \]

\[ \alpha_k = \alpha_{cal} - \alpha_m \]
\[ = \arctan\left(\frac{I_d}{I_q}\right) \]

\[ \begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} I_M \sin(\alpha_{cal} + \alpha_k - \alpha_{cal}) \\ \frac{3}{2} I_M \cos(\alpha_{cal} + \alpha_k - \alpha_{cal}) \end{bmatrix} \]
\[ = \begin{bmatrix} -\frac{3}{2} I_M \sin(0) \\ \frac{3}{2} I_M \cos(0) \end{bmatrix} \]
\[ = \begin{bmatrix} 0 \\ \frac{3}{2} I_M \end{bmatrix} \]

(10.41)

It means that the modified \((d, q)\) transformation matrix in (10.37) can be used to fulfill the required conditions in order to create the input current vectors, when the phase of voltages and currents are assumed to be the same.

However, the phase of currents is not always the same as the phase of voltages, because of the phase impedance of the machine, which also depends on other factors e.g. the machine frequency.
10.6 Stationary Frame Conversion

There are two possibilities to convert the signal in three phase frame to the stationary frame, i.e. in (10.42) by three signals and in (10.43) by two signals, respectively. The calculation in (10.43) has been done based on the assumption in (10.44).

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}}
\end{bmatrix} \begin{bmatrix}
I_U \\
I_V \\
I_W
\end{bmatrix}
\quad (10.42)
\]

\[
\begin{bmatrix}
I_\alpha \\
I_\beta
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}}
\end{bmatrix} \begin{bmatrix}
I_U \\
I_V \\
I_W
\end{bmatrix} = \frac{1}{\sqrt{6}} \left( -\sqrt{3}I_V + \sqrt{3}(-I_U - I_V) \right)
\]

\[
= \frac{1}{\sqrt{6}} \left( 2I_U - I_V - I_W \right)
\]

\[
= \frac{1}{\sqrt{6}} \left( -\sqrt{3}I_U - 2\sqrt{3}I_V \right)
\]

\[
= \frac{1}{\sqrt{6}} \left( 2I_U + I_U \right)
\]

\[
= \frac{-\sqrt{3}}{\sqrt{6}} I_U - \frac{2\sqrt{3}}{\sqrt{6}} I_V
\]

\[
= \frac{3}{\sqrt{6}} I_U
\]

\[
= \frac{-1}{\sqrt{2}} I_U - \sqrt{2}I_V
\]

\[
= \frac{\sqrt{2}}{\sqrt{2}} I_U
\]

\[
= \frac{\sqrt{3}}{\sqrt{2}} I_U
\]

\[
I_U + I_V + I_W = 0
\quad (10.44)
\]